Prestressed Concrete- Basic Concept

Structural Engineering

Prestressed Concrete- Introduction

▸ Concrete is strong in compression, but weak in tension: its tensile strength varies from *8 to 14* percent of its compressive strength.

▸ Due to such a low tensile capacity, flexural cracks develop at early stages of loading.

▸ In order to reduce or prevent cracks from developing, a concentric or eccentric force is imposed in the longitudinal direction of the structural element.

▸ This force prevents the cracks from developing by eliminating or considerably reducing the tensile stresses at the critical mid span and support sections at service load, thereby raising the bending, shear, and torsional capacities of the sections.

▸

Introduction

▶ The sections are then able to behave elastically, and almost the full capacity of the concrete in compression can be efficiently utilized across the entire depth of the concrete sections when all loads act on the structure..

▶ *Such an imposed longitudinal force is called a **prestressing force**, i.e., a compressive force that prestresses the sections along the span of the structural element prior to the application of the transverse gravity dead and live loads or transient horizontal live loads*

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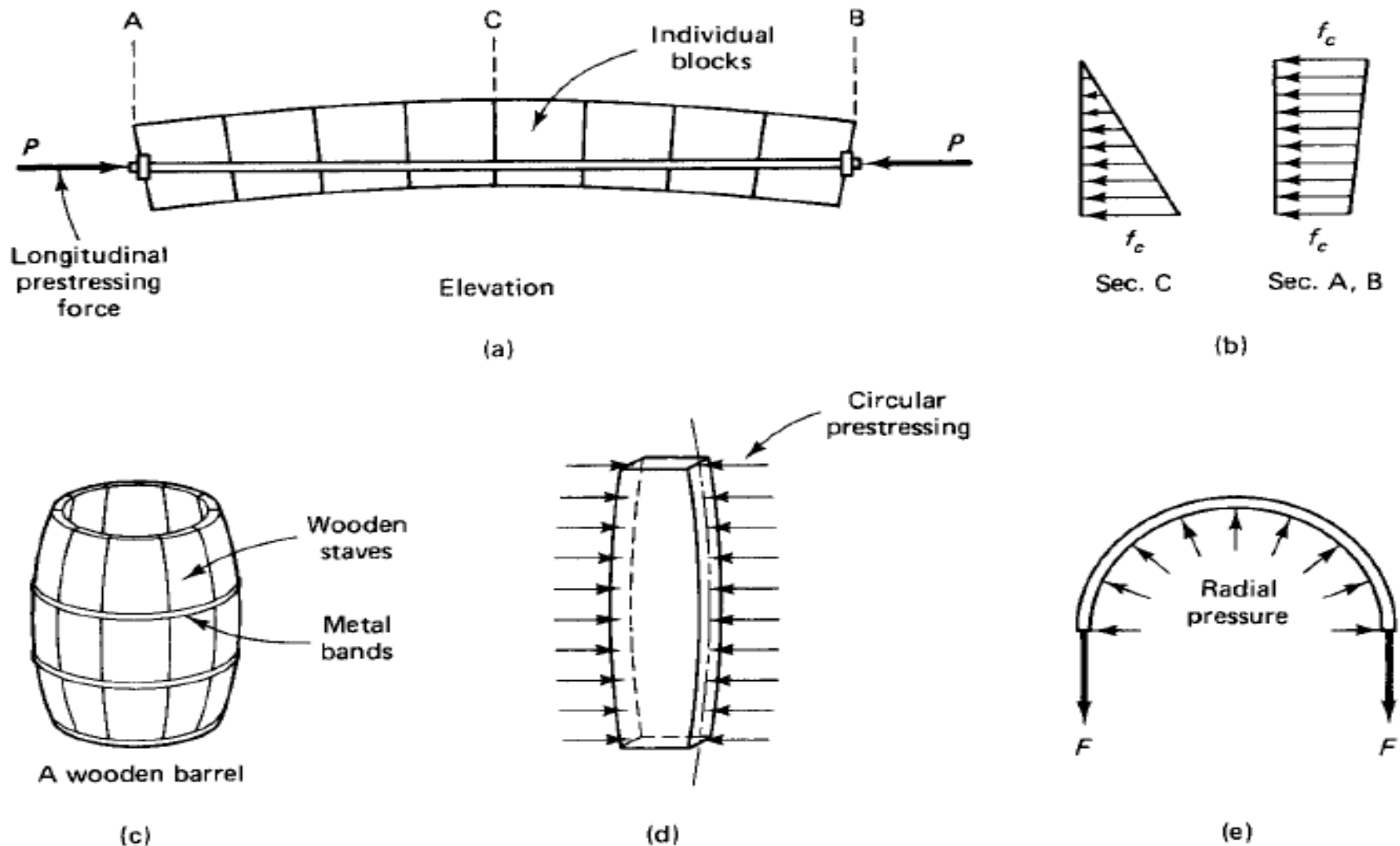
Figure 1.1 Prestressing principle in linear and circular prestressing. (a) Linear prestressing of a series of blocks to form a beam. (b) Compressive stress on midspan section C and end section A or B. (c) Circular prestressing of a wooden barrel by tensioning the metal bands. (d) Circular hoop prestress on one wooden stave. (e) Tensile force F on half of metal band due to internal pressure, to be balanced by circular hoop prestress.

Comparison with Reinforced Concrete

Prestressed Concrete	Reinforced Concrete
• stresses in the prestressed structural member are created before the full dead and live loads are applied, in order to eliminate or considerably reduce the net tensile stresses caused by these loads	• it is assumed that the tensile strength of the concrete is negligible and disregarded. This is because the tensile forces resulting from the bending moments are resisted by the bond created in reinforcement process.
• Cracking and deflection are recoverable before the application of service load	• Cracking and deflection are irrecoverable in RC once the member has reached its limit state at service load.
•The steel required to produce the prestressing force in the prestressed member actively preloads the member.	•The reinforcement in the reinforced concrete member does not exert any force of its own on the member

Comparison with Reinforced Concrete

Prestressed Concrete	Reinforced Concrete
• By controlling the amount of prestress, a structural system can be made either flexible or rigid without influencing its strength.	• In reinforced concrete, such a flexibility in behavior is considerably more difficult to achieve if considerations of economy are to be observed in the design

ECONOMICS OF PRESTRESSED CONCRETE

▶ Pre-stressed members are shallower in depth than their reinforced concrete counterparts for the same span and loading conditions.

▶ In general, the depth of a pre-stressed concrete member is usually about 65 to 80 percent of the depth of the equivalent reinforced concrete member. Hence, the pre-stressed member requires less concrete and about 20 to 35 percent of the amount of reinforcement.

▶ Unfortunately. this saving in material weight is balanced by the higher cost of the higher quality materials needed in pre-stressing.

▶

ECONOMICS OF PRESTRESSED CONCRETE

▸ Also, regardless of the system used, pre-stressing operations themselves result in an added cost:

▸ Form work is more complex, since the geometry of pre-stressed sections is usually composed of flanged sections with thin webs.

▸ In spite of these additional costs, if a large enough number of precast units are manufactured, the difference between at least the initial costs of pre-stressed and reinforced concrete systems is usually not very large.

▸ And the indirect savings are quite substantial, because less maintenance is needed, a longer working life is possible due to better quality control of concrete.

▸

ECONOMICS OF PRESTRESSED CONCRETE

▸ Once the beam span of reinforced concrete exceeds to 90 feet. the dead weight of the beam becomes excessive. resulting in heavier members and. consequently, greater

▸ long-term deflection and cracking.

▸ Thus for larger spans pre-stressed concrete becomes mandatory since arches are expensive to construct and do not perform as well due to the severe long term shrinkage and creep they undergo.

▸ Very large spans such as segmental bridges or cable-stayed bridges can only be constructed through the use of pre-stressing.

▸

Material for Prestressing

- *High strength steel* may not efficiently be used in ordinary reinforced concrete structures, as it will produce excessive cracking.

- Modulus of elasticity of high-strength steel is almost the same as that of ordinary steel.

- Very high strains are produced in high strength steel close to its yield strength and hence cause wide and excessive cracking in adjoining concrete.

- The cracking reduces moment of inertia, increases deflection, reduces shear strength and causes corrosion of steel reinforcement.

- The appearance of the members is also disturbed due to cracking and structure above the member under consideration may also be damaged due to large deflections

Material for Prestressing

- Use of high strength concrete in ordinary reinforced concrete structures is not much beneficial as only a smaller part of the section on compression side is subjected to maximum compressive stresses.

- In greater part of the section, compressive strength is not fully utilized and the tensile strength of concrete is neglected altogether after the first cracking.

- In prestressed concrete members, larger portion of the cross-section is subjected to compressive stresses and high strength of concrete is effectively utilized.
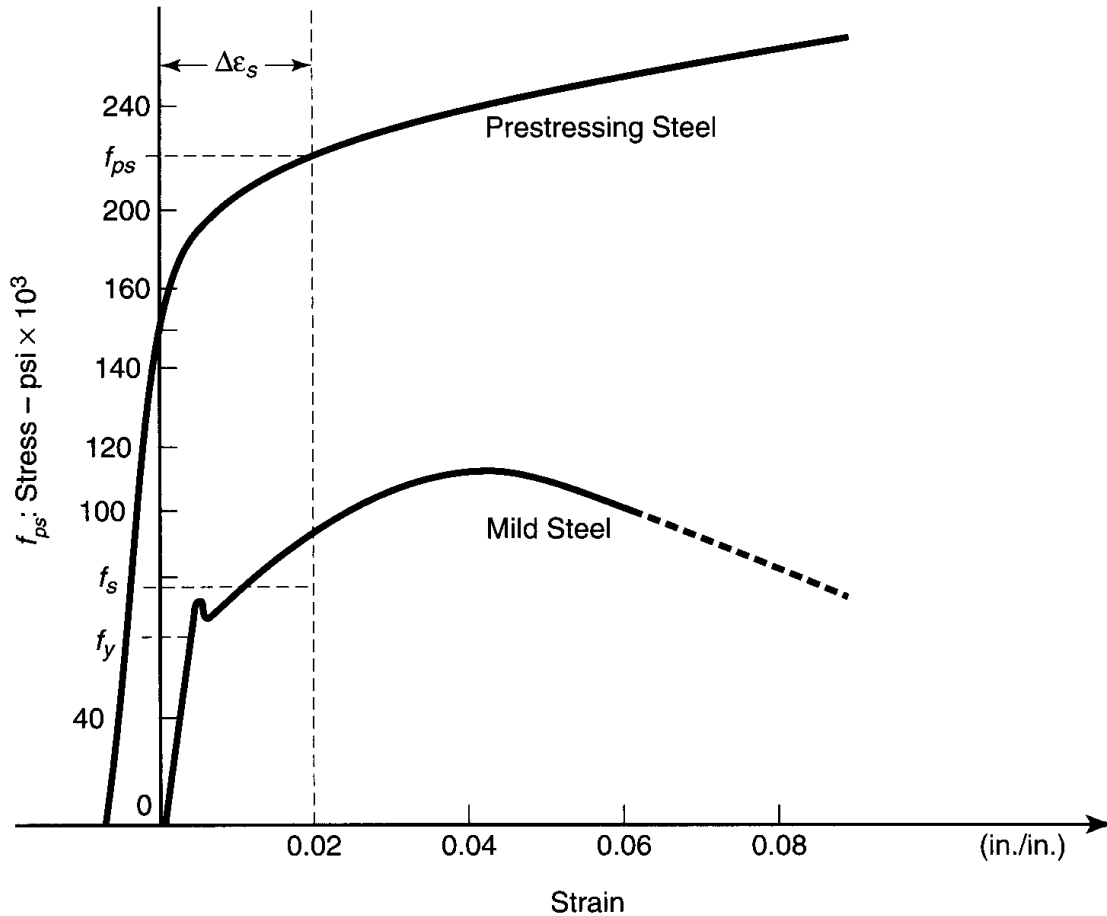
Material for Prestressing

- Ordinary steels, such as Grade 280 steel, are ineffective in providing effective prestressing.

- Grade 280 steel is used to prestress a beam and is stretched to a stress of $0.94f_y$ = 263MPa. The corresponding steel strain is ε_s = 263/200000 = 0.00132.

- In case the steel bars are anchored, there will be some slip in the anchorage and some elastic shortening of the member will take place reducing the strain by approximately 0.0007.

- Steel strain just after transfer will be approximately 0.00132 – 0.0007 = 0.00062 and about half of the prestressing force will be lost almost suddenly.
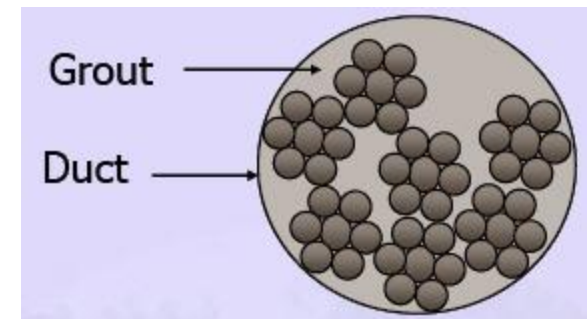
Material for Prestressing

▶ The most commonly used steel for pretenstioned concrete is in the form of a seven-wire, uncoated, stress-relived strand having a minimum tensile strength (fpu) of 250,000 psi or 270,000 psi.

▶ „Prestressing steel does not exhibit the definite yield point characteristic found in the normal ductile steel used in reinforcing steel .

▶ The yield strength for prestressing wire and strand is a "specific yield strength" that is obtained from the stress-strain diagram at 1% strain, according to ASTM.

▶ Nevertheless, the specified yield point is not as important in prestressing steel as is the yield point of ductile steel.

▶ „It is a consideration, however, when determining the ultimate strength of a beam

▶

Material for Prestressing

Figure 2.18(b) Stress-Strain Diagram for Prestressing Steel Strands in Comparison with Mild Steel Bar Reinforcement.

Strand

Tendon

Material for Prestressing

- ***Higher strength concrete***, having $f_c{}' \geq 35$ MPa, is usually used for prestressed members for quick and efficient construction with lesser loss of prestressing force due to elastic shortening, creep and shrinkage.

- The advantages of high strength concrete in prestressed construction are as follows:

  1. With larger compressive strength of concrete, its modulus of elasticity is increased reducing the elastic shortening due to prestress force. Further, long term deformations due to creep and shrinkage are also reduced. Hence, by the use of high strength concrete, the prestress losses are significantly reduced increasing the efficiency of such construction.

  2. The concrete gains high early strength and hence the prestress may be applied to the concrete earlier. The speed of construction is increased when high strength concrete is used.

Material for Prestressing

3. Bearing strength is better for high strength concrete. This makes it easy to transfer the prestress forces at the anchorages, which may require lesser contact area of anchorage fittings.

4. The bond between steel and concrete is improved when high strength concrete is used.

5. Higher strengths are easy to achieve in case of prestressed precast construction in factories where batching, mixing, placing and curing is carried out under controlled conditions.

- Following notation is used to specify strength of concrete for prestressed construction:

  - f_c' = the specified minimum 28-day cylinder compressive strength of concrete.

  - f_{ci}' = the minimum cylinder compressive strength of concrete at the time of initial transfer of prestressing force.

  - The allowable stresses in concrete are different at various stages of loading

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Type of Prestressing

▸ Prestressed concrete structures can be classified in a number of ways depending upon the feature of designs and constructions.

1. Pre-tensioning: In which the tendons are tensioned before the concrete is placed, tendons are temporarily anchored and tensioned and the prestress is transferred to the concrete after it is hardened.
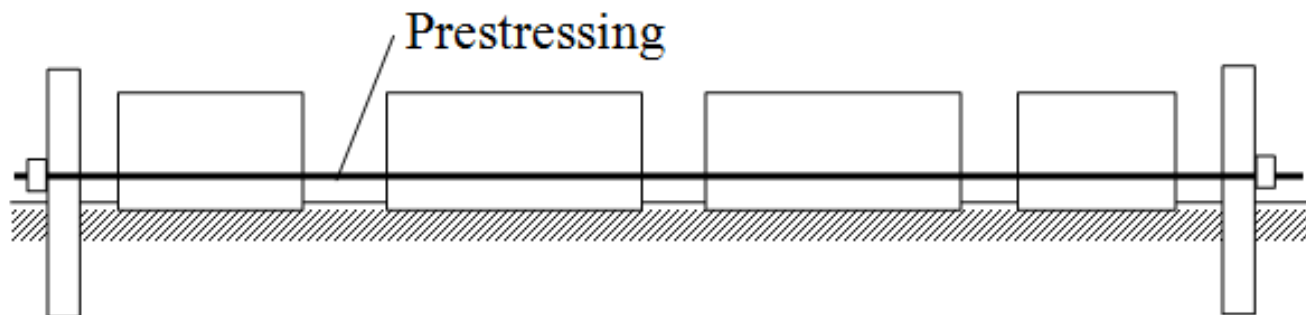

Fig. 19.10. Prestressing Bed for Pre-Tensioning.

Type of Prestressing

1. <u>Post-tensioning</u>: In which the tendon is tensioned after concrete has hardened. Tendons are placed in sheathing at suitable places in the member before casting and later after hardening of concrete.

   Plain concrete beams or beams with some ordinary reinforcement are cast with embedded plastic or metal tubes forming ducts for the tendons to be placed later.
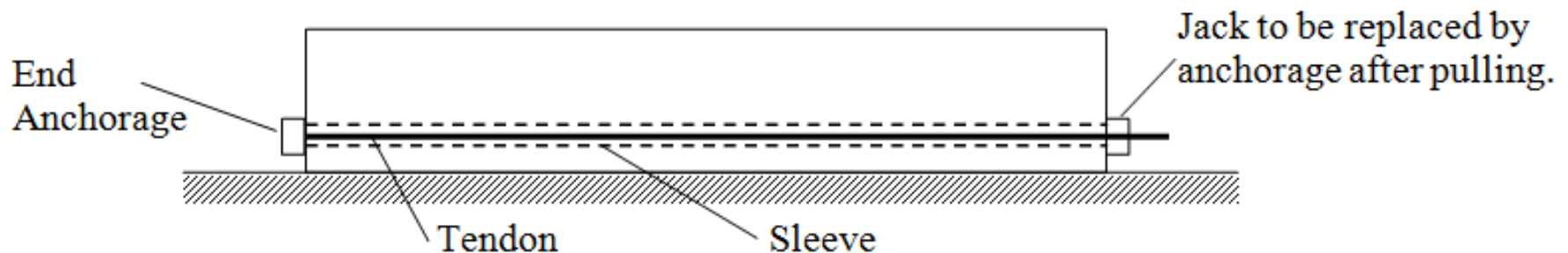

Fig. 19.11. Beam Subjected to Post-Tensioning.

LOSS OF PRESTRESS

▸ ***Jacking force***(P_j) is the force required to pull the tendon to required stress level.

▸ ***Initial prestress force*** (P_i) is the force after transfer and is equal to jacking force reduced due to the instantaneous shortening of the concrete.

▸ ***Effective prestress force*** (P_e) is the amount of prestress force left after instantaneous and time dependent losses.

▸ ***Effectiveness ratio*** (R) is defined as the ratio of effective prestress to the initial prestress.

$$R = P_e / P_i$$

▸

LOSS OF PRESTRESS

- A usually conservative value of $R = 0.85$ may be used in place of detailed actual calculations.

- A better estimate of losses may be made by ***Time-Step Method*** where losses are calculated for the required time by dividing it into a number of intervals.

- For each time interval, losses are estimated with respect to starting and ending of the interval.

- This approach reduces the inter-dependence of various losses on one another

LOSS OF PRESTRESS

- Total loss of prestress may be divided into two categories as under:

- *a) Instantaneous Losses:* These are the losses due to anchorage set, friction between the tendon and concrete and elastic shortening of concrete.

- Following notation may be used to represent these losses:

  ▸ Δf_{PF} = loss due to friction, MPa

  ▸ Δf_{PA} = loss due to anchorage set, MPa

  ▸ Δf_{PE} = loss due to elastic shortening, MPa

b) *Time Dependent Losses:*

- These are the losses due to creep and shrinkage of concrete and relaxation of steel and are denoted by the following symbols:

  - ▸ Δf_{PSR} = loss due to shrinkage of concrete, MPa
  - ▸ Δf_{PCR} = loss due to creep of concrete, MPa
  - ▸ Δf_{PR} = loss due to relaxation of steel, MPa

- Total prestress loss (Δf_{PT}) for the two methods of prestressing may be written as under:

  - ▸ i) *Pre-tensioned members:*$\Delta f_{PT} = \Delta f_{PES} + \Delta f_{PSR} + \Delta f_{PCR} + \Delta f_{PR}$
  - ▸ ii) *Post-tensioned members:*$\Delta f_{PT} = \Delta f_{PF} + \Delta f_{PA} + \Delta f_{PES} + \Delta f_{PSR} + \Delta f_{PCR} + \Delta f_{PR}$

▸

BASIC CONCEPTS OF PRESTRESSING

▸ The prestressing force P that satisfies the particular conditions of geometry and loading of a given element (see Figure 1.2) is determined from the principles of mechanics and of stress-strain relationships.

▸ Sometimes simplification is necessary, as when a prestressed beam is assumed to be homogeneous and elastic.

▸ Consider, then, a simply supported rectangular beam subjected to a *concentric prestressing* force *P as shown in Figure 1.2(a). The compressive stress on the beam cross section* is uniform.

BASIC CONCEPTS OF PRESTRESSING

Figure 1.2 Concrete fiber stress distribution in a rectangular beam with straight tendon. (a) Concentric tendon, prestress only. (b) Concentric tendon, self-weight added. (c) Eccentric tendon, prestress only. (d) Eccentric tendon, self-weight added.

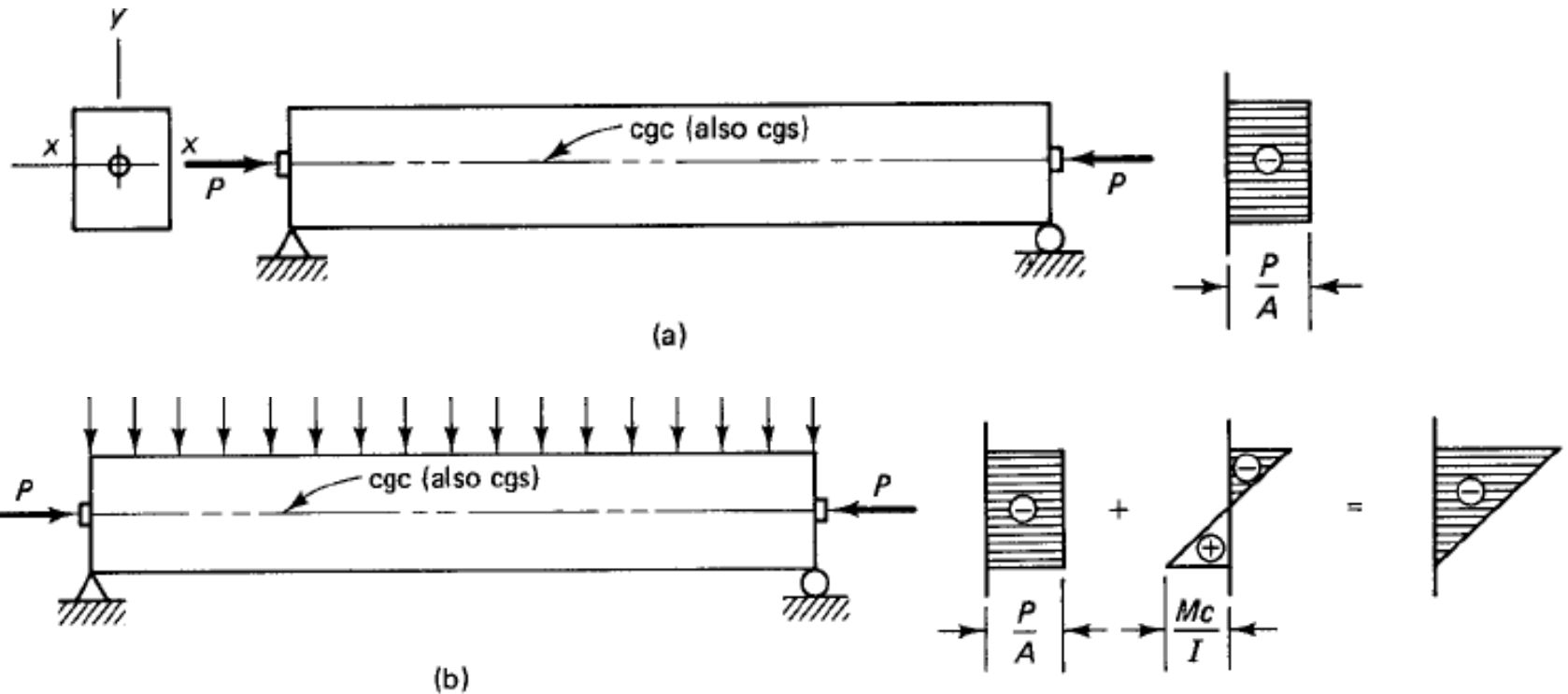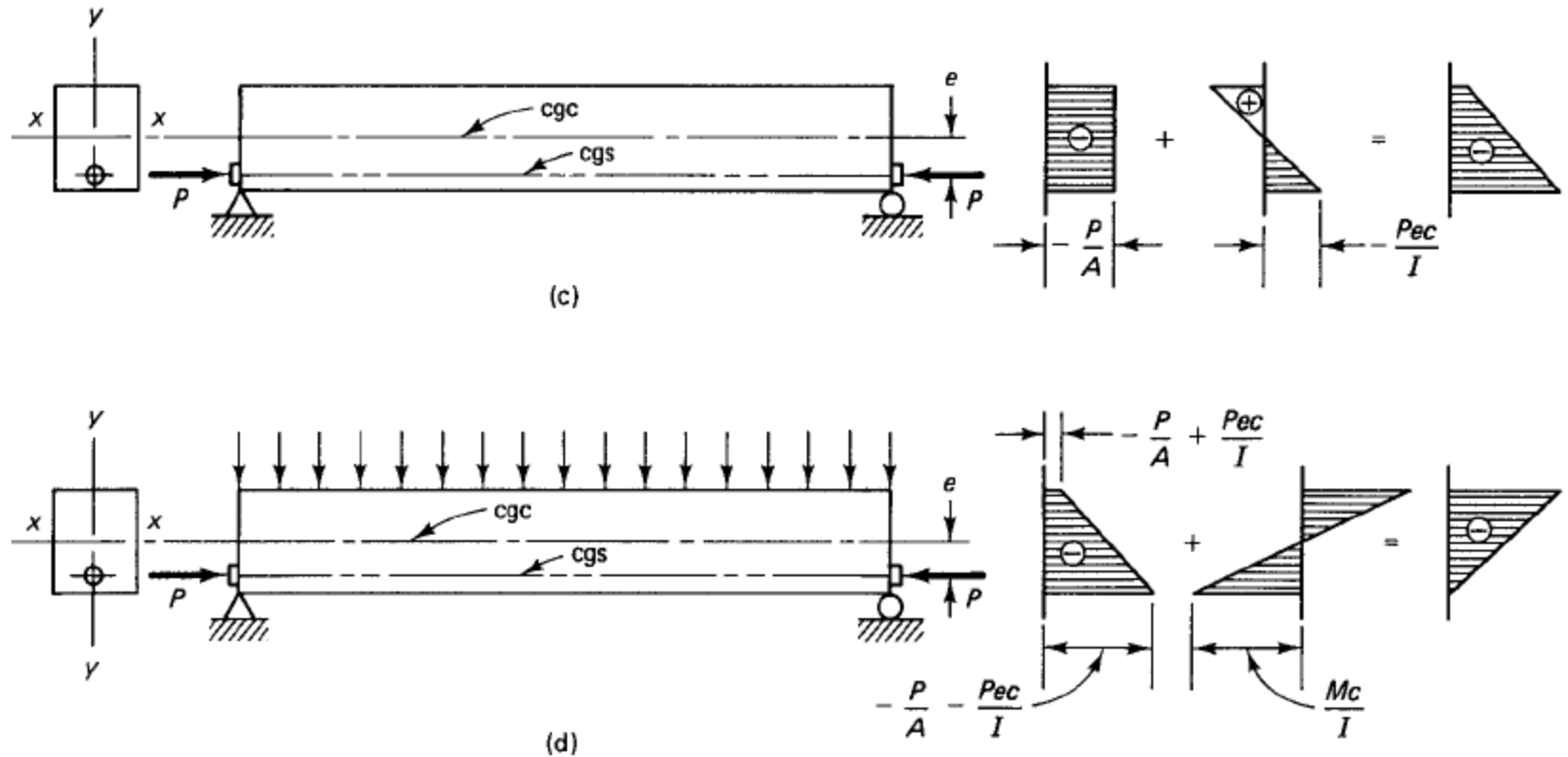# BASIC CONCEPTS OF PRESTRESSING

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BASIC CONCEPTS OF PRESTRESSING

The compressive stress on the beam cross section is uniform and has an intensity

$$f = -\frac{P}{A_c} \tag{1.1}$$

where $A_c = bh$ is the cross-sectional area of a beam section of width b and total depth h. A *minus* sign is used for compression and a *plus* sign for tension throughout the text. Also, bending moments are drawn on the tensile side of the member.

If external transverse loads are applied to the beam, causing a maximum moment M at midspan, the resulting stress becomes

$$f^t = -\frac{P}{A} - \frac{Mc}{I_g} \tag{1.2a}$$

and

$$f_b = -\frac{P}{A} + \frac{Mc}{I_g} \tag{1.2b}$$

where f^t = stress at the top fibers
f_b = stress at the bottom fibers
$c = \frac{1}{2}h$ for the rectangular section
I_g = gross moment of inertia of the section ($bh^3/12$ in this case)

BASIC CONCEPTS OF PRESTRESSING

▸ Equation 1.2b indicates that the presence of prestressing-compressive stress − *P/A is reducing* the tensile flexural stress *Mc/I to the extent intended in the design, either elimi*nating tension totally (even inducing compression), or permitting a level of tensile stress within allowable code limits.

▸ The section is then considered uncracked and behaves elastically: the concrete's inability to withstand tensile stresses is effectively compensated for by the compressive force of the prestressing tendon.

▸

BASIC CONCEPTS OF PRESTRESSING

▸ The compressive stresses in Equation 1.2a at the top fibers of the beam due to prestressing are compounded by the application of the loading stress − *Mc/I, as seen in Figure*

▸ Hence, the compressive stress capacity of the beam to take a substantial external load is reduced by the *concentric prestressing force.*

▸ In order to avoid this limitation, the prestressing tendon is placed *eccentrically below the neutral axis at midspan, to* induce tensile stresses at the top fibers due to prestressing.

BASIC CONCEPTS OF PRESTRESSING

▸ If the tendon is placed at eccentricity *e from the center of gravity of the concrete, termed the cgc line, it creates a moment Pe, and the ensuing stresses at midspan become*

$$f^t = -\frac{P}{A_c} + \frac{Pec}{I_g} - \frac{Mc}{I_g} \qquad (1.3a)$$

$$f_b = -\frac{P}{A_c} - \frac{Pec}{I_g} + \frac{Mc}{I_g} \qquad (1.3b)$$

Since the support section of a simply supported beam carries no moment from the external transverse load, high tensile fiber stresses at the top fibers are caused by the eccentric prestressing force. To limit such stresses, the eccentricity of the prestressing tendon profile,the *cgs line, is made less at the support section than at the midspan section, or eliminated* altogether, or else a negative eccentricity above the cgc line is used.

▸

Basic Concept Method

▸ In the **basic concept method** of designing prestressed concrete elements, the concrete fiber stresses are *directly computed from the external forces applied to the concrete by* longitudinal prestressing and the external transverse load.

▸ Equations 1.3a and b can be modified and simplified for use in calculating stresses at the initial prestressing stage and at service load levels.

▸ If *Pi is the initial prestressing force before stress losses, and Pe is the* effective prestressing force after losses, then ϒ can be defined as the residual prestress factor

$$\gamma = \frac{P_e}{P_i} \qquad\qquad (1.3c)$$

▸

Basic Concept Method

can be defined as the residual prestress factor. Substituting r^2 for I_g/A_c in Equations 1.3, where r is the radius of gyration of the gross section, the expressions for stress can be rewritten as follows:

(a) Prestressing Force Only

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) \tag{1.4a}$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) \tag{1.4b}$$

where c_t and c_b are the distances from the center of gravity of the section (the cgc line) to the extreme top and bottom fibers, respectively.

Basic Concept Method

(b) Prestressing Plus Self-weight

If the beam self-weight causes a moment M_D at the section under consideration, Equations 1.4a and b, respectively, become

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \qquad (1.5a)$$

and

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} \qquad (1.5b)$$

where S^t and S_b are the moduli of the sections for the top and bottom fibers, respectively.
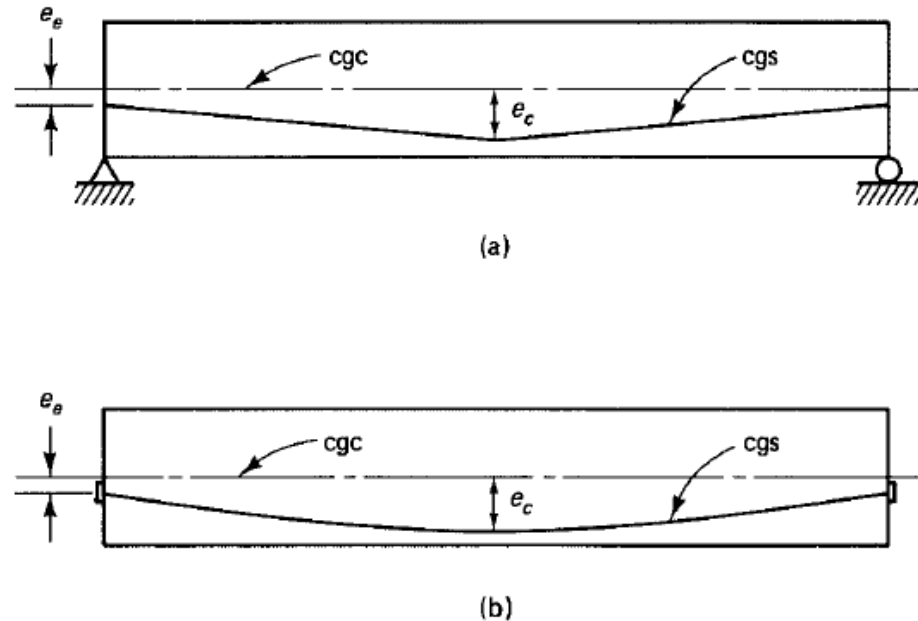
Basic Concept Method

Figure 1.3 Prestressing tendon profile. (a) Harped tendon. (b) Draped tendon.

▶ Figure 1.3(a) shows a harped profile usually used for pretensioned beams and for concentrated transverse loads.

▶ Figure 1.3(b) shows a draped tendon usually used in post-tensioning.

▶

Basic Concept Method

▶ The change in eccentricity from the midspan to the support section is obtained by raising the prestressing tendon either abruptly from the midspan to the support, a process called harping, or gradually in a parabolic form, a process called draping.

Basic Concept Method

Subsequent to erection and installation of the floor or deck, live loads act on the structure, causing a superimposed moment M_s. The full intensity of such loads normally occurs after the building is completed and some time-dependent losses in prestress have already taken place. Hence, the prestressing force used in the stress equations would have to be the effective prestressing force P_e. If the total moment due to gravity loads is M_T, then

$$M_T = M_D + M_{SD} + M_L \qquad (1.6)$$

where M_D = moment due to self-weight

M_{SD} = moment due to superimposed dead load, such as flooring

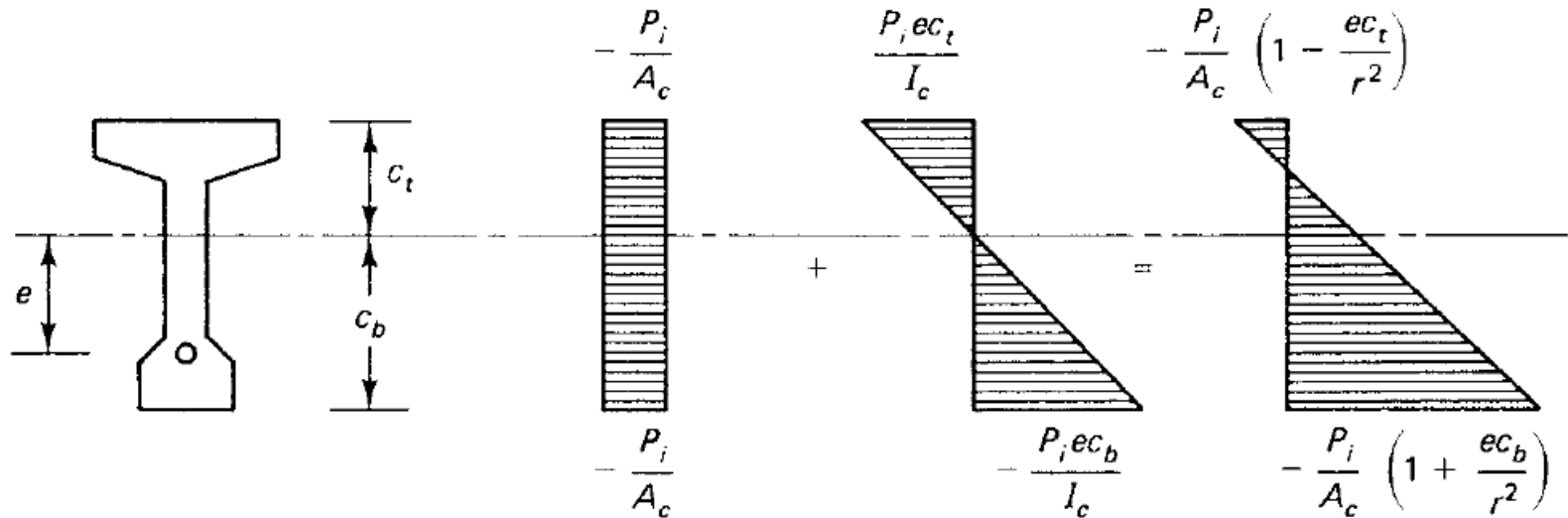M_L = moment due to live load, including impact and seismic loads if any

Equations 1.5 then become

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t} \qquad (1.7a)$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b} \qquad (1.7b)$$
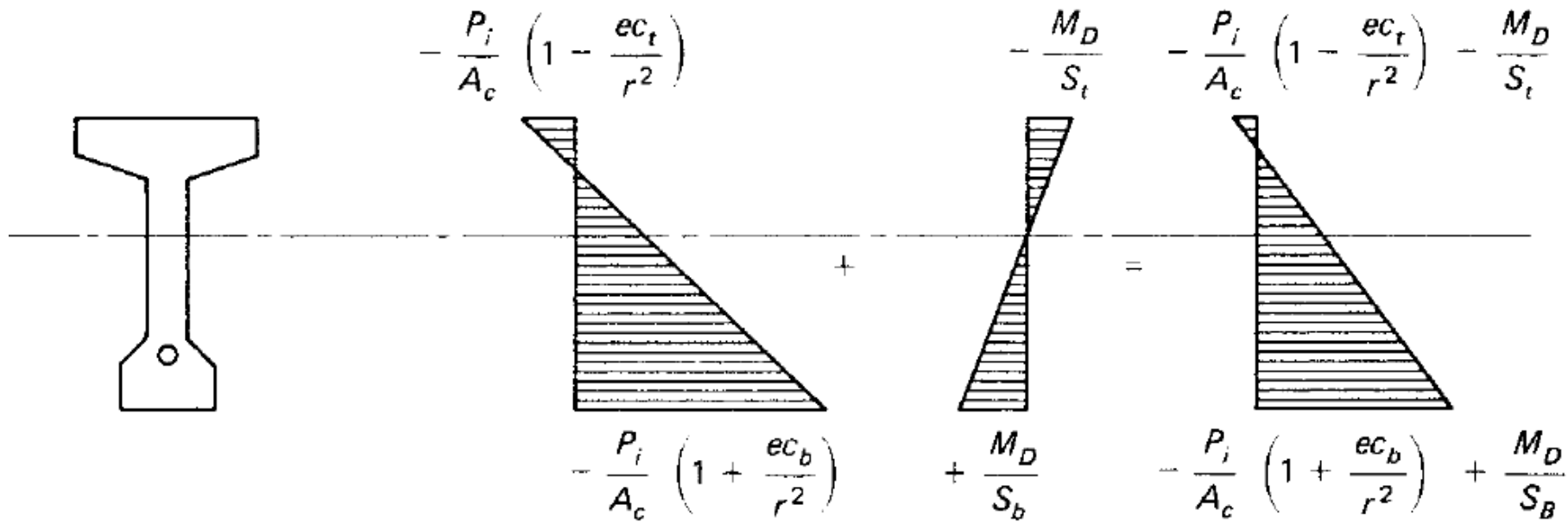
▶

Basic Concept Method

Some typical elastic concrete stress distributions at the critical section of a prestressed flanged section are shown in Figure 1.4. The tensile stress in the concrete in part (c) permitted at the extreme fibers of the section cannot exceed the maximum permissible in the code, e.g., $f_t = 6\sqrt{f_c'}$ at midspan in the ACI code. If it is exceeded, bonded nonprestressed reinforcement proportioned to resist the total tensile force has to be provided to control cracking at service loads.

$$-\frac{P_i}{A_c} \qquad \frac{P_i ec_t}{I_c} \qquad -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right)$$

$$-\frac{P_i}{A_c} \qquad -\frac{P_i ec_b}{I_c} \qquad -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right)$$

(a)

Figure 1.4 Elastic fiber stresses due to the various loads in a prestressed beam. (a) Initial prestress before losses. (b) Addition of self-weight. (c) Service load at effective prestress.

$$-\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right)$$

$$-\frac{M_D}{S_t}$$

$$-\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S_t}$$

$$-\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right)$$

$$+\frac{M_D}{S_b}$$

$$-\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_B}$$

(b)

Figure 1.4 Elastic fiber stresses due to the various loads in a prestressed beam. (a) Initial prestress before losses. (b) Addition of self-weight. (c) Service load at effective prestress.

Basic Concept Method

$$-\frac{P_e}{A_c}\left(1-\frac{ec_t}{r^2}\right)-\frac{M_D}{S_t} \qquad -\frac{M_{SD}+M_L}{S_t} \qquad -\frac{P_e}{A_c}\left(1-\frac{ec_t}{r^2}\right)-\frac{M_T}{S_t}$$

$$-\frac{P_e}{A_c}\left(1+\frac{ec_b}{r^2}\right)+\frac{M_D}{S_b} \qquad +\frac{M_{SD}+M_L}{S_b} \qquad -\frac{P_e}{A_c}\left(1+\frac{ec_b}{r^2}\right)+\frac{M_T}{S_b}$$
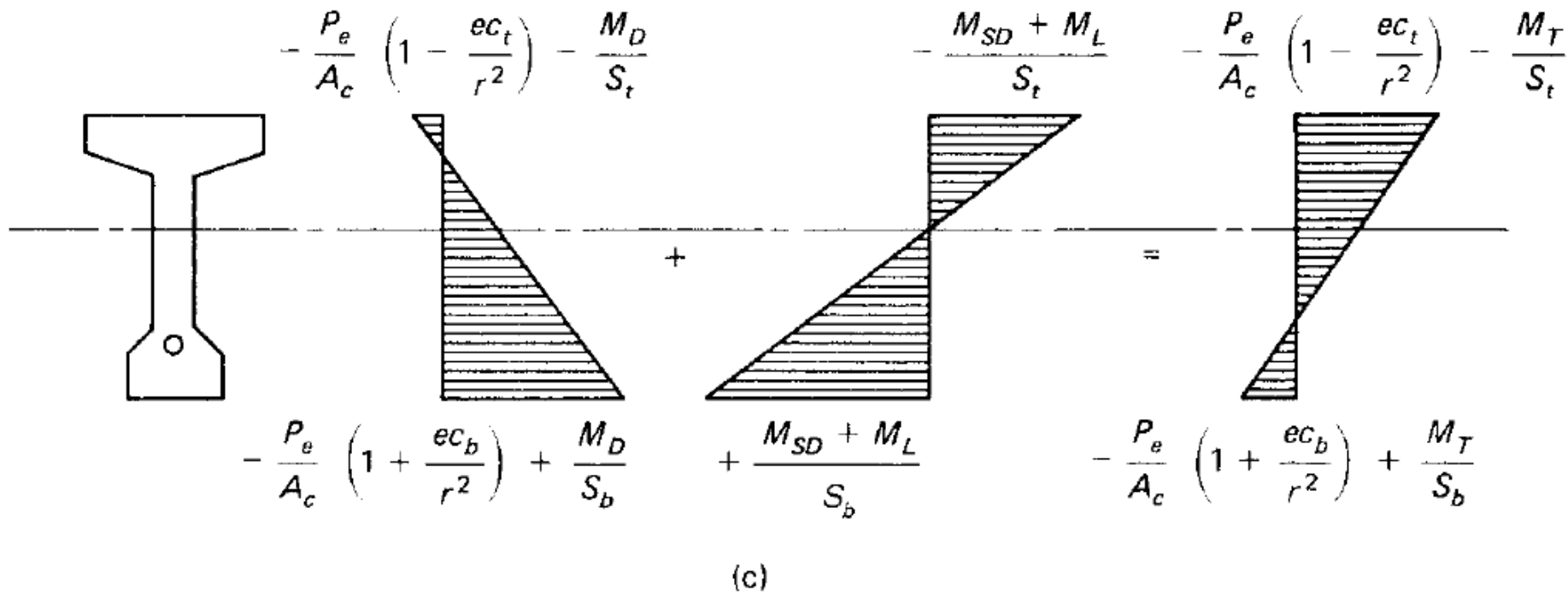
(c)

Figure 1.4 Elastic fiber stresses due to the various loads in a prestressed beam. (a) Initial prestress before losses. (b) Addition of self-weight. (c) Service load at effective prestress.

Basic Concept Method

$$-\frac{P_e}{A_c}\left(1-\frac{ec_t}{r^2}\right)-\frac{M_D}{S_t}$$

$$-\frac{M_{SD}+M_L}{S_t}$$

$$-\frac{P_e}{A_c}\left(1-\frac{ec_t}{r^2}\right)-\frac{M_T}{S_t}$$

$$-\frac{P_e}{A_c}\left(1+\frac{ec_b}{r^2}\right)+\frac{M_D}{S_b}$$

$$+\frac{M_{SD}+M_L}{S_b}$$

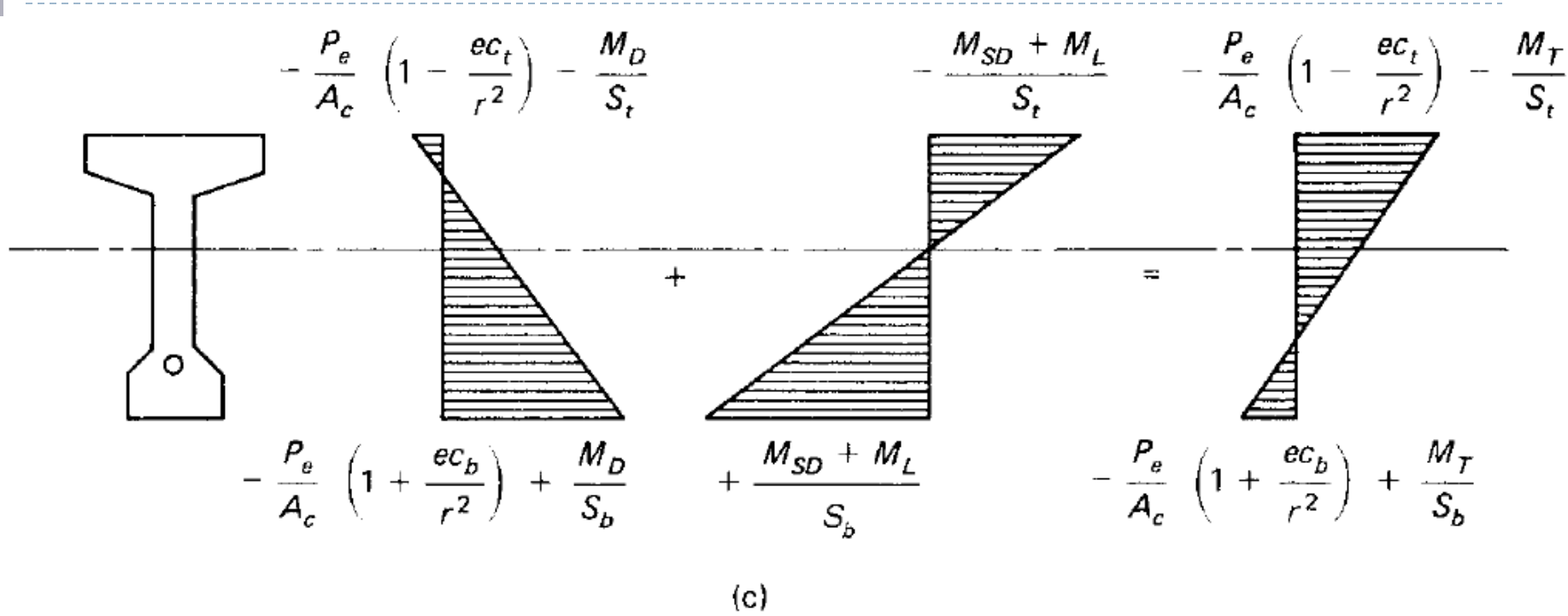$$-\frac{P_e}{A_c}\left(1+\frac{ec_b}{r^2}\right)+\frac{M_T}{S_b}$$

(c)

Figure 1.4 Elastic fiber stresses due to the various loads in a prestressed beam. (a) Initial prestress before losses. (b) Addition of self-weight. (c) Service load at effective prestress.

ACI Maximum Stress Limits

ACI MAXIMUM PERMISSIBLE STRESSES IN CONCRETE AND REINFORCEMENT

Following are definitions of some important mathematical terms used in this section:

f_{py} = specified yield strength of prestressing tendons, in psi

f_y = specified yield strength of nonprestressed reinforcement, in psi

f_{pu} = specified tensile strength of prestressing tendons, in psi

f_c' = specified compressive strength of concrete, in psi

f_{ci}' = compressive strength of concrete at time of initial prestress

2.8.1 Concrete Stresses in Flexure

Stresses in concrete immediately after prestress transfer (before time-dependent pre-stress losses) shall not exceed the following:

(a) Extreme fiber stress in compression . . . . $0.60\,f_{ci}'$

(b) Extreme fiber stress in tension except as permitted in (c) . . . . $3\sqrt{f_{ci}'}$

(c) Extreme fiber stress in tension at ends of simply supported members . . . . $6\sqrt{f_{ci}'}$

ACI Maximum Stress Limits

Where computed tensile stresses exceed these values, bonded auxiliary reinforcement (nonprestressed or prestressed) shall be provided in the tensile zone to resist the total tensile force in concrete computed under the assumption of an uncracked section.

Stresses in concrete at service loads (after allowance for all prestress losses) shall not exceed the following:

(a) Extreme fiber stress in compression due to prestress plus sustained load, where sustained dead load and live load are a large part of the total service load .... $0.45 f'_c$

(b) Extreme fiber stress in compression due to prestress plus total load, if the live load is transient .... $0.60 f'_c$

(c) Extreme fiber stress in tension in precompressed tensile zone.......... $6\sqrt{f'_c}$

(d) Extreme fiber stress in tension in precompressed tensile zone of members (except two-way slab systems), where analysis based on transformed cracked sections and on bilinear moment-deflection relationships shows that immediate and long-time deflections comply with the ACI definition requirements and minimum concrete cover requirements .... $12\sqrt{f'_c}$

2.8.2 Prestressing Steel Stresses

Tensile stress in prestressing tendons shall not exceed the following:

(a) Due to tendon jacking force .... $0.94 f_{py}$
but not greater than the lesser of $0.80 f_{pu}$ and the maximum value recommended by the manufacturer of prestressing tendons or anchorages.

ACI Maximum Stress Limits

(b) Immediately after prestress transfer. . . . . $0.82\,f_{py}$
but not greater than $0.74\,f_{pu}$.

(c) Post-tensioning tendons, at anchorages and couplers, immediately after
tendon anchorage . . . . $0.70\,f_{pu}$

Example 1.1

A pretensioned simply supported 10LDT24 double T-beam without topping has a span of 64 ft (19.51 m) and the geometry shown in Figure 1.11. It is subjected to a uniform super-imposed gravity dead-load intensity W_{SD} and live-load intensity W_L summing to 420 plf (6.13 kN/m). The initial prestress before losses is $f_{pi} \cong 0.70 f_{pu} = 189{,}000$ psi (1,303 MPa), and the effective prestress after losses is $f_{pe} = 150{,}000$ psi (1,034 MPa). Compute the extreme fiber stresses at the midspan due to

- **(a)** the initial full prestress and no external gravity load
- **(b)** the final service load conditions when prestress losses have taken place.

$$c_b = 17.77 \text{ in. (452 mm)}$$

$$c_t = 6.23 \text{ in. (158 mm)}$$

$$e_c = 14.77 \text{ in. (375 mm)}$$

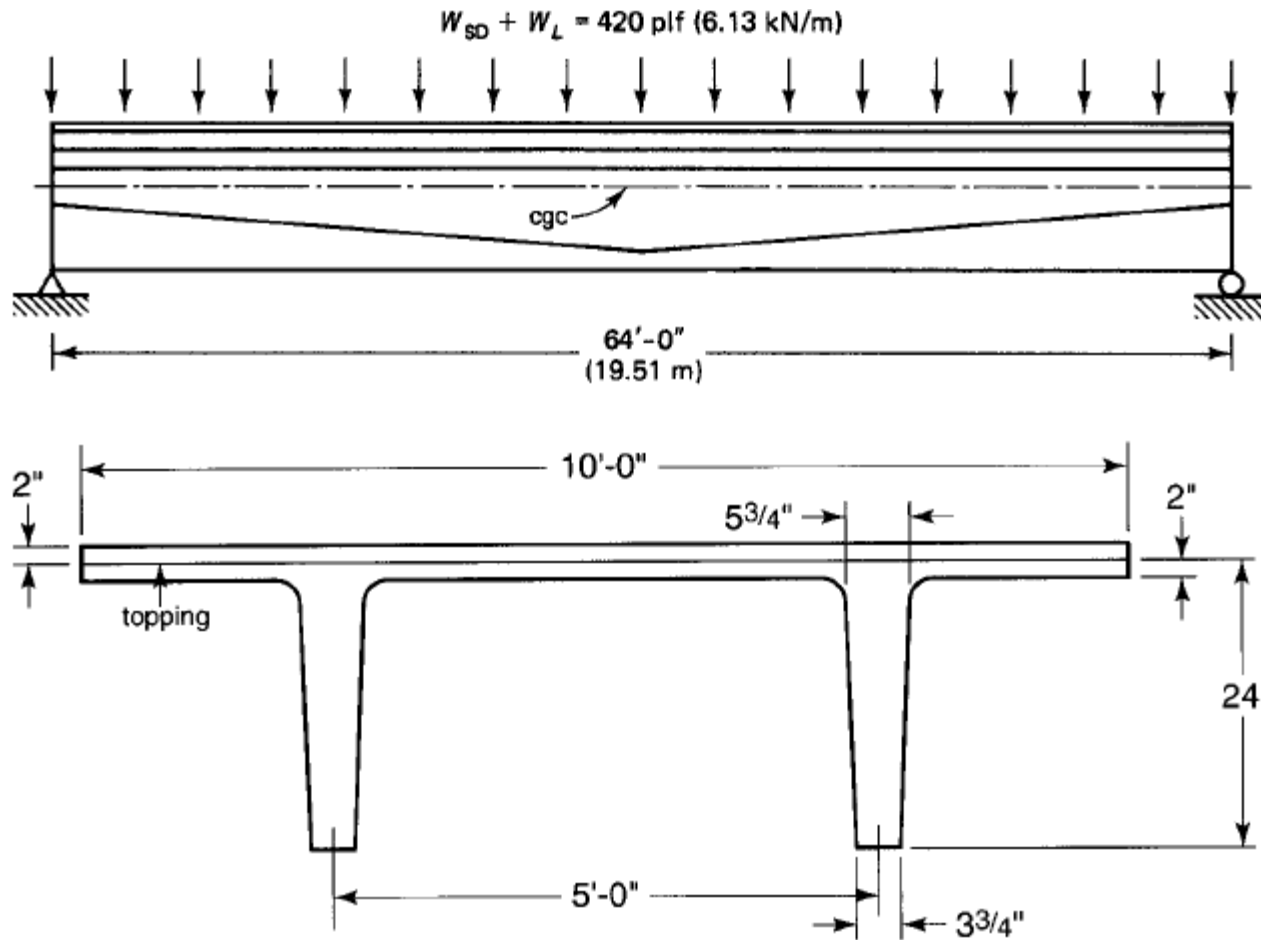$$e_e = 7.77 \text{ in. (197 mm)}$$

$$S_b = 1{,}264 \text{ in.}^3 \text{ (20,714 cm}^3)$$

$$S^t = 3{,}607 \text{ in.}^3 \text{ (59,108 cm}^3)$$

$$W_D = 359 \text{ plf (4.45 kN/m)}$$

Example

Example

Allowable stress data are as follows:

$f'_c = 6,000$ psi, lightweight (41.4 MPa)

$f_{pu} = 270,000$ psi, stress relieved (1.862 MPa) = specified tensile strength of the tendons

$f_{py} = 220,000$ psi (1.517 MPa) = specified yield strength of the tendons

$f_{pe} = 150,000$ psi (1,034 MPa)

$f_t = 12 \sqrt{f'_c} = 930$ psi (6.4 MPa) = maximum allowable tensile stress in concrete

$f'_{ci} = 4,800$ psi (33.1 MPa) = concrete compressive strength at time of initial prestress

$f_{ci} = 0.6 \ f'_{ci} = 2,880$ psi (19.9 MPa) = maximum allowable stress in concrete at initial prestress

$f_c = 0.45 \ f'_c$ = maximum allowable compressive stress in concrete at service

Assume that ten $\frac{1}{2}$in.-dia. Seven-wire-strand (ten 12.7-mm-dia strand) tendons with a 108-D1 strand pattern are used to prestress the beam.

$$A_c = 449 \text{ in.}^2 \ (2,915 \text{ cm}^2)$$

$$I_c = 22,469 \text{ in.}^4 \ (935,347 \text{ cm}^4)$$

$$r^2 = I_c/A_c = 50.04 \text{ in.}^2$$

▶

Example

$$c_b = 17.77 \text{ in. (452 mm)}$$

$$c_t = 6.23 \text{ in. (158 mm)}$$

$$e_c = 14.77 \text{ in. (375 mm)}$$

$$e_e = 7.77 \text{ in. (197 mm)}$$

$$S_b = 1,264 \text{ in.}^3 \ (20,714 \text{ cm}^3)$$

$$S^t = 3,607 \text{ in.}^3 \ (59,108 \text{ cm}^3)$$

$$W_D = 359 \text{ plf } (4.45 \text{ kN/m})$$

(i) *Initial Conditions at Prestressing*

$$A_{ps} = 10 \times 0.153 = 1.53 \text{ in.}^2$$

$$P_i = A_{ps}f_{pi} = 1.53 \times 189,000 = 289,170 \text{ lb } (1,287 \text{ kN})$$

$$P_e = 1.53 \times 150,000 = 229,500 \text{ lb } (1,020 \text{ kN})$$

The midpan self-weight dead-load moment is

$$M_D = \frac{wl^2}{8} = \frac{359\,(64)^2}{8} \times 12 = 2,205,696 \text{ in.-lb. } (249 \text{ kN-m})$$

From Equations 1.5 and 1.7,

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t}$$

$$= -\frac{289,170}{449}\left(1 - \frac{14.77 \times 6.23}{50.04}\right) - \frac{2,205,696}{3,607}$$

$$= +540.3 - 611.5 \cong -70 \text{ psi } (C)$$

From Equations 1.5 and 1.7,

$$f^t = -\frac{P_i}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t}$$

$$= -\frac{289{,}170}{449}\left(1 - \frac{14.77 \times 6.23}{50.04}\right) - \frac{2{,}205{,}696}{3{,}607}$$

$$= +540.3 - 611.5 \cong -70 \text{ psi } (C)$$

$$f_b = -\frac{P_i}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b}$$

$$= -\frac{289{,}170}{449}\left(1 + \frac{14.77 \times 17.77}{50.04}\right) + \frac{2{,}205{,}696}{1{,}264}$$

$$= -4{,}022.1 + 1{,}745.0 \cong -2{,}277 \text{ psi } (C)$$

$$\leq f_{ci} = -2{,}880 \text{ psi allowed, O.K.}$$

(ii) Final Condition at Service Load Midspan moment due to superimposed dead and live load is

$$M_{SD} + M_L = \frac{420\,(64)^2}{8} \times 12 = 2{,}580{,}480 \text{ in.-lb}$$

Total Moment $M_T = 2{,}205{,}696 + 2{,}580{,}480$

$$= 4{,}786{,}176 \text{ in.-lb. } (541 \text{ kN-m})$$

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{229{,}500}{449}\left(1 - \frac{14.77 \times 6.23}{50.04}\right) - \frac{4{,}786{,}176}{3{,}607}$$

$$= +429 - 1{,}327 \cong -898 \text{ psi } (C)\,(7 \text{ MPa})$$

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t}$$

$$= -\frac{229{,}500}{449}\left(1 - \frac{14.77 \times 6.23}{50.04}\right) - \frac{4{,}786{,}176}{3{,}607}$$

$$= +429 - 1{,}327 \cong -898 \text{ psi } (C) \text{ (7 MPa)}$$

$$< f_c = 0.45 \times 6{,}000 = 2{,}700 \text{ psi, O.K.}$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b}$$

$$= -\frac{229{,}500}{449}\left(1 + \frac{14.77 \times 17.77}{50.04}\right) + \frac{4{,}786{,}176}{1{,}264}$$

$$= -3{,}192 + 3{,}786 \cong +594 \text{ psi } (T) \text{ (5.2 MPa)}$$

$$< f_t = 12\sqrt{f_c'} = 930 \text{ psi, O.K.}$$

Design of Prestressed Members

▶ Flexural stresses are the result of external, or imposed, bending moments.

▶ They control the selection of the geometrical dimensions of the pre-stressed concrete section regardless of whether it is pre-tensioned or post-tensioned.

▶ The design process starts with the choice of a preliminary geometry, and by trial and adjustment it converges to a final section with geometrical details of the concrete cross section and the sizes and alignments of the pre-stressing strands.

Design of Prestressed Members

▶ The section satisfies the flexural (bending) requirements of concrete stress and steel stress limitations. Thereafter, other factors such as shear and torsion capacity, deflection, and cracking are analyzed and satisfied.

▶ The principles and methods presented in Chapter 1 for service load computations are extended into step-by-step procedures for the design of pre-stressed concrete linear elements, taking into consideration the impact of the magnitude of pre-stress losses discussed earlier.

▶

Design of Prestressed Members

▸ Note that a logical sequence in the design process entails first the service-load design of the section required in flexure, and then the analysis of the available moment strength Mn of the section for the limit state at failure. Throughout the book, a negative sign (-) is used to denote compressive stress and a positive sign (+) is used to denote tensile stress in the concrete section.

▸ A convex or hogging shape indicates negative bending moment; a concave or sagging shape denotes positive bending moment, as shown in Figure 4.1.
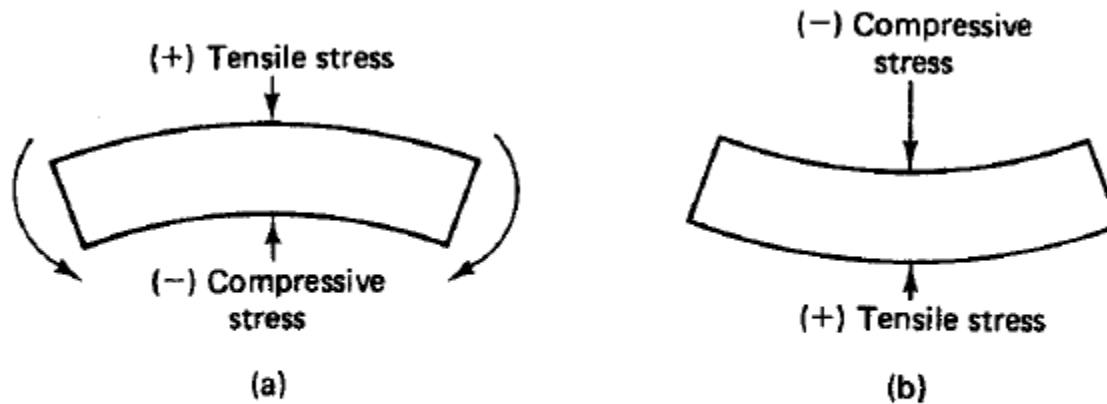
▸

Design of Prestressed Members

Figure 4.1 Sign convention for flexure stress and bending moment. (a) Negative bending moment. (b) Positive bending moment.

Design of Prestressed Members

▶ Unlike the case of reinforced concrete members, the external dead load and partial live load are applied to the pre-stressed concrete member at varying concrete strengths at various loading stages. These loading stages can be summarized as follows:

▶ Initial prestress force P, is applied; then, at transfer, the force is transmitted from the prestressing strands to the concrete.

▶ The full self-weight WD acts on the member together with the initial prestressing force, provided that the member is simply supported, i.e., there is no intermediate support.

▶

Design of Prestressed Members

▸ The full superimposed dead load WSD including topping for composite action, is applied to the member.

▸ Most short-term losses in the prestressing force occur, leading to a reduced pre-stressing force Pe.

▸ The member is subjected to the full service load, with long-term losses due to creep, shrinkage, and strand relaxation taking place and leading to a net pre-stressing force Pe.

▸ Overloading of the member occurs under certain conditions up to the limit state at failure.

▸

Design of Prestressed Members

▶ A typical loading history and corresponding stress distribution across the depth of the critical section are shown in Figure 4.2.

▶ Load vs deformation is shown in the figure 4.3 for various loading stages.

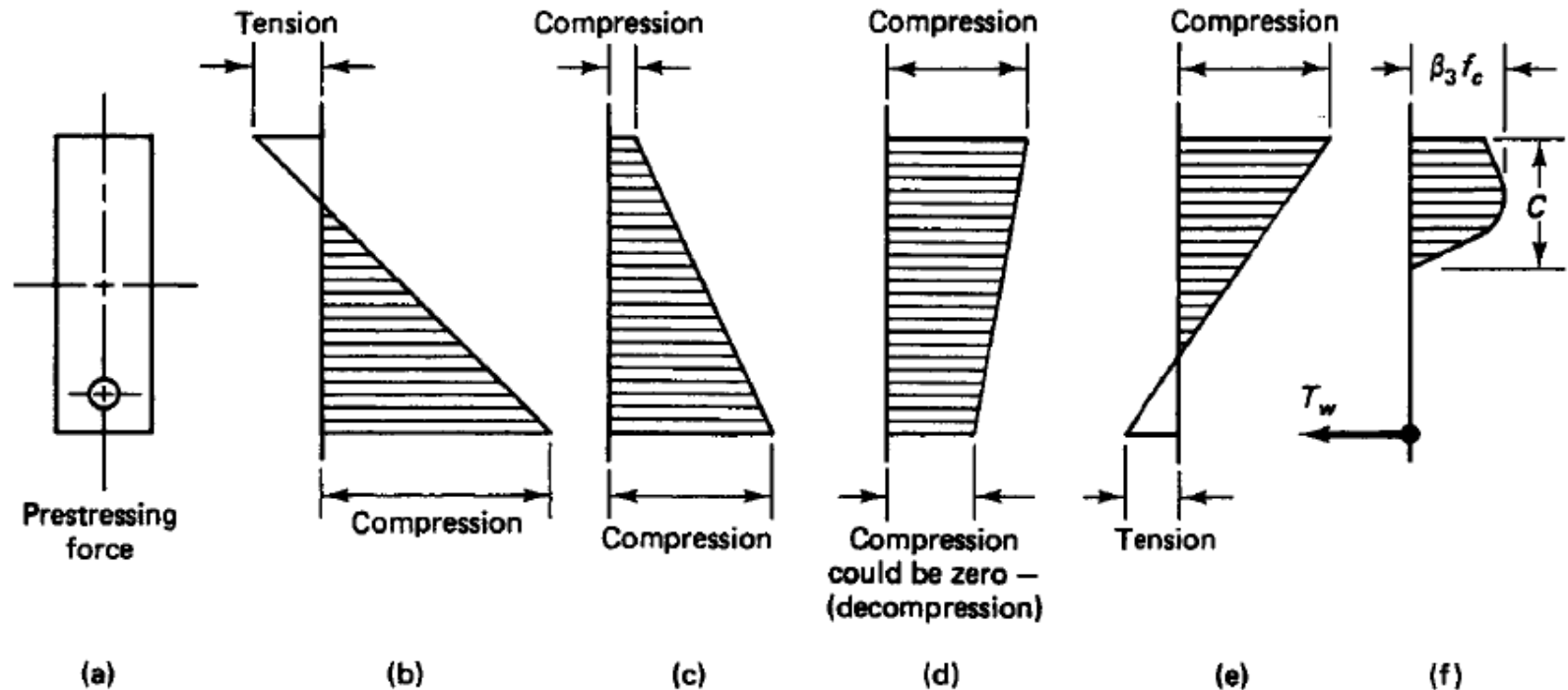# Design of Prestressed Members

Figure 4.2 Flexural stress distribution throughout loading history. (a) Beam section. (b) Initial prestressing stage. (c) Self-weight and effective prestress. (d) Full dead load plus effective prestress. (e) Full service load plus effective prestress. (f) Limit state of stress at ultimate load for underreinforced beam.
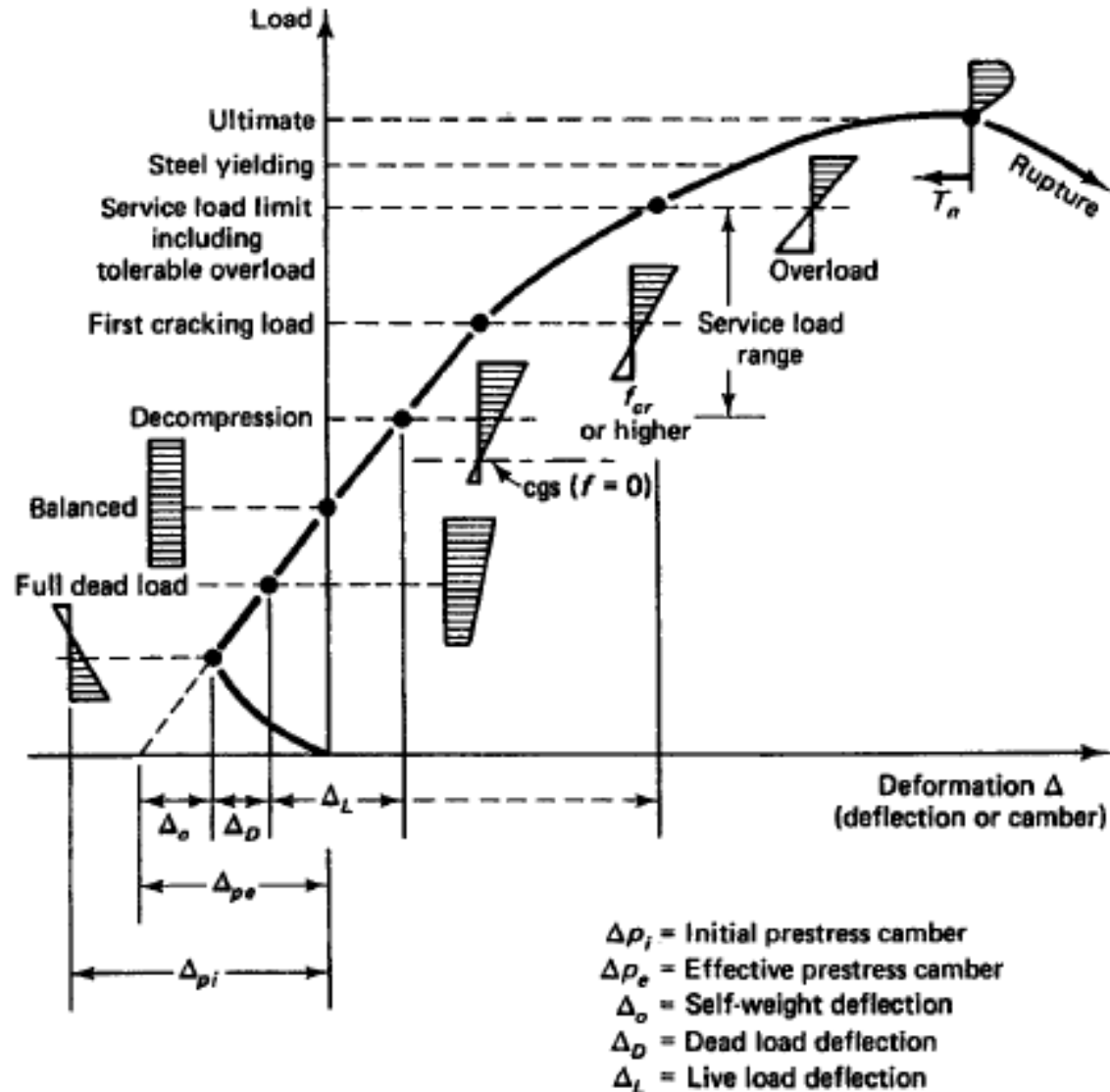
Design of Prestressed Members

Figure 4.3 Load-deformation curve of typical prestressed beam.

Design of Prestressed Members

SELECTION OF GEOMETRICAL PROPERTIES OF SECTION COMPONENTS

General Guidelines

▸ Under service-load conditions, the beam is assumed to be homogeneous and elastic.

▸ Since it is also assumed (because expected) that the prestress compressive force transmitted to the concrete closes the crack that might develop at the tensile fibers of the beam, beam sections are considered uncracked.

▸ Stress analysis of prestressed beams under these conditions is no different from stress analysis of a steel beam, or, more accurately, a beam column.

▸

Design of Prestressed Members

▸ The axial force due to prestressing is always present regardless of whether bending moments do or do not exist due to other external or self-loads.

▸ It is advantageous to have the alignment of the pre-stressing tendon eccentric at the critical sections, such as the mid span section in a simple beam and the support section in a continuous beam.

▸ As compared to a rectangular solid section, a nonsymmetrical flanged section has the advantage of efficiently using the concrete material and of concentrating the concrete in the compressive zone of the section where

▸ it is most needed.

▸

Design of Prestressed Members

Minimum Section Modulus

▸ To design or choose the section, a determination of the required minimum section modulus, S_b and S^t has to be made first. If following stresses are met.

Design of Prestressed Members

f_{ci} = maximum allowable compressive stress in concrete immediately after transfer and prior to losses

$= 0.60 f'_{ci}$

f_{ti} = maximum allowable tensile stress in concrete immediately after transfer and prior to losses

$= 3\sqrt{f'_{ci}}$ (the value can be increased to $6\sqrt{f'_{ci}}$ at the supports for simply supported members)

f_c = maximum allowable compressive stress in concrete after losses at service-load level

$= 0.45 f'_c$ or $0.60 f'_c$ when allowed by the code

f_t = maximum allowable tensile stress in concrete after losses at service load level

$= 6\sqrt{f'_c}$ (the value can be increased in one-way systems to $12\sqrt{f'_c}$ if long-term deflection requirements are met)

Design of Prestressed Members

Effective Stresses after Losses

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_D}{S^t} \le f_t \qquad (4.2a)$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_D}{S_b} \le f_c \qquad (4.2b)$$

Service-load Final Stresses

$$f^t = -\frac{P_e}{A_c}\left(1 - \frac{ec_t}{r^2}\right) - \frac{M_T}{S^t} \le f_c \qquad (4.3a)$$

$$f_b = -\frac{P_e}{A_c}\left(1 + \frac{ec_b}{r^2}\right) + \frac{M_T}{S_b} \le f_t \qquad (4.3b)$$

Design of Prestressed Members

where $M_T = M_D + M_{SD} + M_L$

P_i = initial prestress

P_e = effective prestress after losses

t denotes the top, and b denotes the bottom fibers

e = eccentricity of tendons from the concrete section center of gravity, cgc

r^2 = square of radius of gyration

S^t/S_b = top/bottom section modulus value of concrete section

Design of Prestressed Members

4.2.2.1 Beams With Variable Tendon Eccentricity. Beams are prestressed with either draped or harped tendons. The maximum eccentricity is usually at the midspan controlling section for the simply supported case. Assuming that the effective prestressing force is

$$P_e = \gamma P_i$$

where γ is the residual prestress ratio, the loss of prestress is

$$P_i - P_e = (1 - \gamma)P_i \tag{a}$$

If the actual concrete extreme fiber stress is equivalent to the maximum allowable stress, the change in this stress after losses, from Equations 4.1a and b, is given by

$$\Delta f^t = (1 - \gamma)\left(f_{ti} + \frac{M_D}{S^t}\right) \tag{b}$$

$$\Delta f_b = (1 - \gamma)\left(-f_{ci} + \frac{M_D}{S_b}\right) \tag{c}$$

From Figure 4.4(a), as the superimposed dead-load moment M_{SD} and live-load moment M_L act on the beam, the net stress at top fibers is

Design of Prestressed Members

From Figure 4.4(a), as the superimposed dead-load moment M_{SD} and live-load moment M_L act on the beam, the net stress at top fibers is

$$f_n^t = f_{ti} - \Delta f^t - f_c$$

or

$$f_n^t = \gamma f_{ti} - (1 - \gamma)\frac{M_D}{S^t} - f_c \qquad \text{(d)}$$

The net stress at the bottom fibers is

$$f_{bn} = f_t - f_{ci} - \Delta f_b$$

or

$$f_{bn} = f_t - \gamma f_{ci} - (1 - \gamma)\frac{M_D}{S_b} \qquad \text{(e)}$$

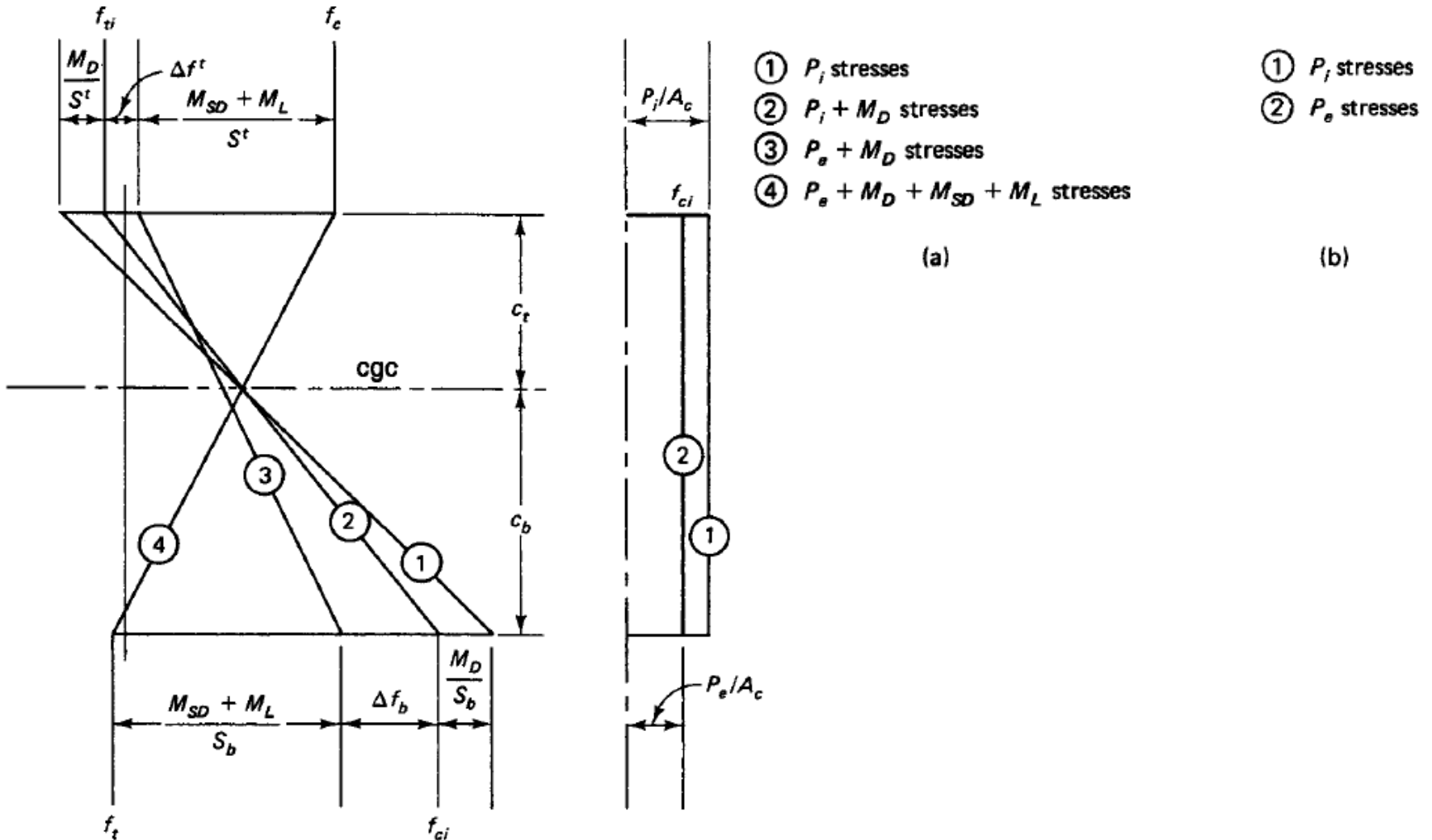# Design of Prestressed Members

Figure 4.4(a) Maximum fiber stresses in beams with draped or harped tendons. (a) Critical section such as midspan. (b) Support section of simply-supported beam ($e_e = 0$ as tendon moves to cgc).

Design of Prestressed Members

From Equations (d) and (e), the chosen section should have section moduli values

$$S^t \geq \frac{(1 - \gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c} \tag{4.4a}$$

and

$$S_b \geq \frac{(1 - \gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}} \tag{4.4b}$$

The required eccentricity of the prestressing tendon at the critical section, such as the midspan section, is

$$e_c = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i} + \frac{M_D}{P_i} \tag{4.4c}$$

where $\bar{f}_{ci}$ is the concrete stress at transfer at the level of the centroid cgc of the concrete section and

$$P_i = \bar{f}_{ci} A_c$$

Design of Prestressed Members

Thus,

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci}) \qquad (4.4d)$$

where tensile stress is $\oplus$ and comprehensive stress is $\ominus$.
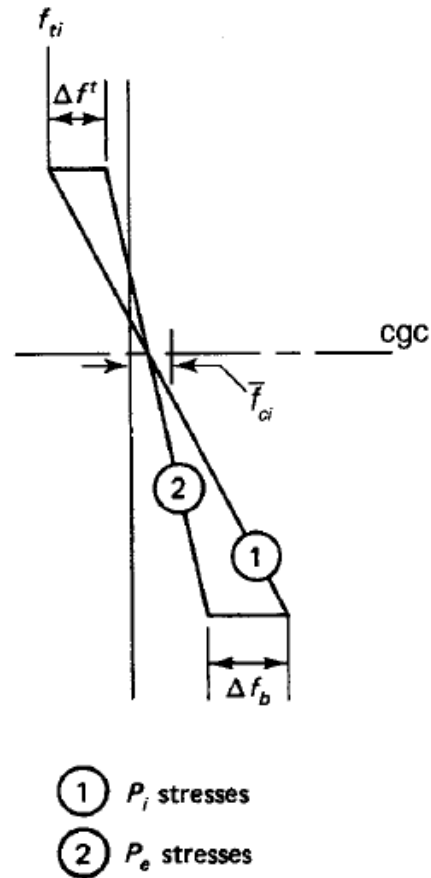
Design of Prestressed Members

Figure 4.4(b) Maximum fiber stresses at support section of beams with straight tendons (stress distribution at midspan section similar to that of Figure 4.4a).

Design of Prestressed Members

4.2.2.2 Beams with Constant Tendon Eccentricity. Beams with constant tendon eccentricity are beams with straight tendons, as is normally the case in precast moderate-span simply supported beams. Because the tendon has a large eccentricity at the support, creating large tensile stresses at the top fibers without any reduction due to superimposed $M_D + M_{SD} + M_L$, in such beams smaller eccentricity of the tendon at midspan has to be used as compared to a similar beam with a draped tendon. In other words, the controlling section is the support section, for which the stress distribution at the support is shown in Figure 4.4(b). Hence,

$$\Delta f^t = (1 - \gamma)(f_{ti}) \tag{a'}$$

and

$$\Delta f_b = (1 - \gamma)(-f_{ci}) \tag{b'}$$

The net stress at the service-load condition after losses at the top fibers is

$$f_n^t = f_{ti} - \Delta f^t - f_c$$

or

$$f_n^t = \gamma f_{ti} - f_{cs} \tag{c'}$$

Design of Prestressed Members

or

$$f_n^t = \gamma f_{ti} - f_{cs} \tag{c'}$$

where f_{cs} is the actual service-load stress in concrete. The net stress at service load after losses at the bottom fibers is

$$f_{bn} = f_t - f_{ci} - \Delta f_b$$

or

$$f_{bn} = f_t - \gamma f_{ci} \tag{d'}$$

From Equations (c) and (d), the chosen section should have section moduli values

Design of Prestressed Members

$$S^t \geq \frac{M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c} \tag{4.5a}$$

and

$$S_b \geq \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}} \tag{4.5b}$$

The required eccentricity value at the critical section, such as the support for an ideal beam section having properties close to those required by Equations 4.5a and b, is

$$e_e = (f_{ti} - \bar{f}_{ci}) \frac{S^t}{P_i} \tag{4.5c}$$

A graphical representation of section moduli of nominal sections is shown in Figure 4.5. It may be used as a speedy tool for the choice of initial trial sections in the design process.
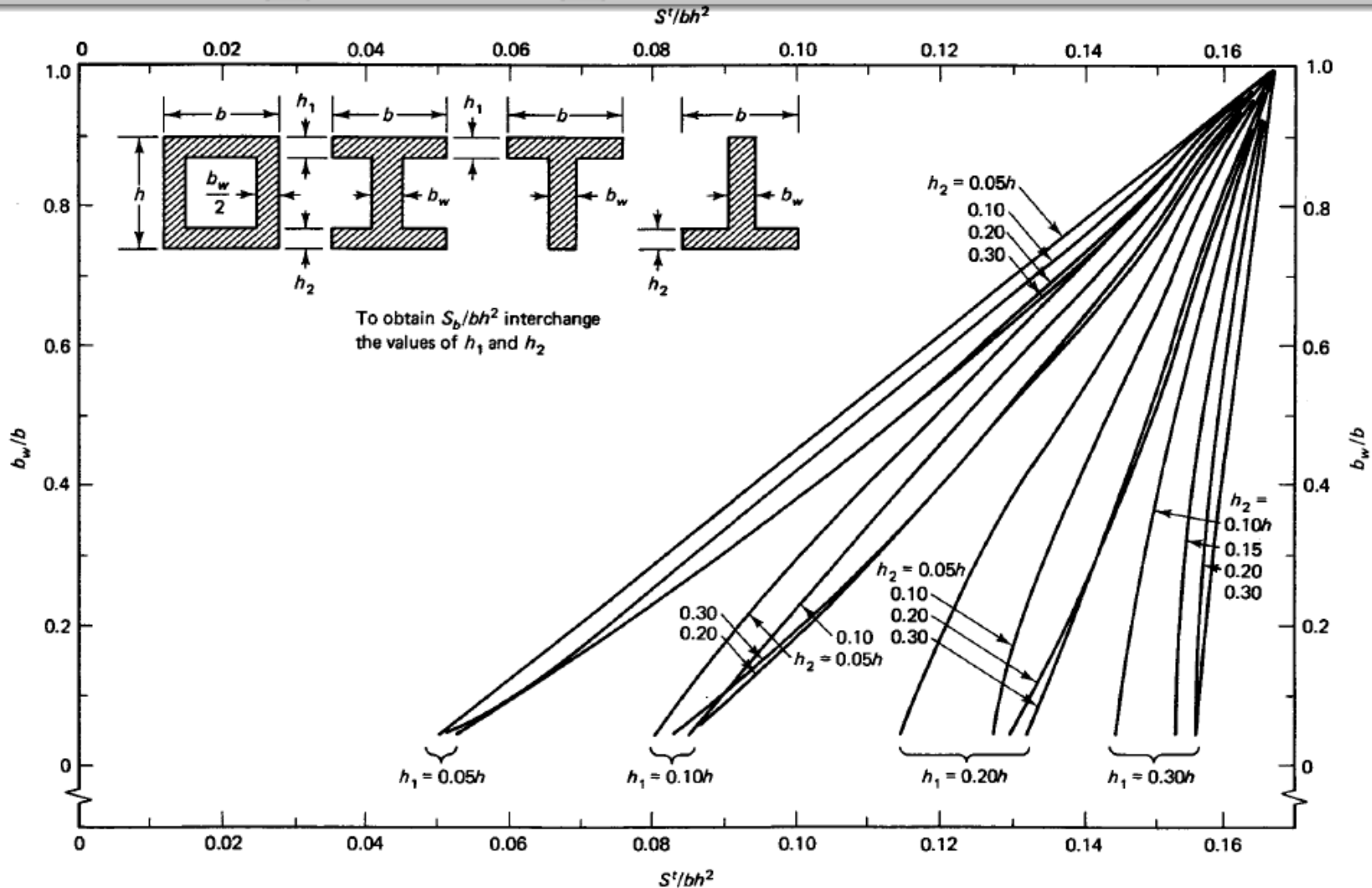
Design of Prestressed Members

Figure 4.5 Section moduli of flanged and boxed sections

PRESTRESSED CONCRETE- Example

- A post-tensioned pre stressed concrete beam is to carry a live load of 1,000 lb/ ft and superimposed dead load of 500 lb/ ft in addition to its own weight, on a 40-ft simple span. Normal density concrete will be used with design strength $fc' = 6,000$ psi. It is estimated that, at the time of transfer, the concrete will have attained 70 percent of its ultimate strength, or 4,200 psi.

- Time-dependent losses may be assumed at 15 percent of the initial prestress, giving an effectiveness ratio of 0.85.

- Determine the required concrete dimensions, magnitude of pre-stress force, and eccentricity of the steel centroid based on ACI stress limitations as given in Tables 3.1 and 3.2.

- (w_l=14.6 KN/m, wd, = 7.3 kN/m, span = 12.2 m, $fc' = 41$ MPa, and fci, = 29 MPa.)

PRESTRESSED CONCRETE- Example

Referring to Table 3.1, we obtain the following stress limits:

$$f_{ci} = -0.60 \times 4{,}200 = -2{,}520 \text{ psi}$$

$$f_{ti} = 3\sqrt{4{,}200} = +195 \text{ psi}$$

$$f_{cs} = -0.45 \times 6{,}000 = -2{,}700 \text{ psi}$$

$$f_{ts} = 6\sqrt{6{,}000} = +465 \text{ psi}$$

The self-weight of the girder will be estimated at 250 plf. The moments due to transverse loading are

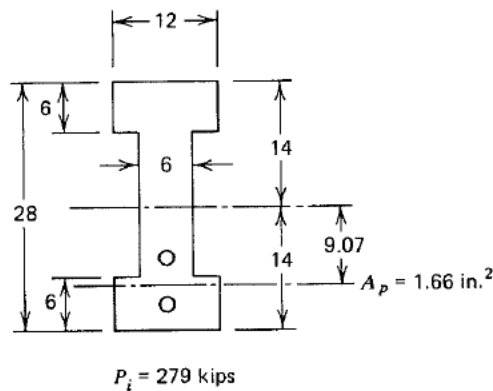$$M_o = \frac{1}{8} \times 0.250 \times 40^2 = 50 \text{ ft-kips}$$

$$M_d + M_l = \frac{1}{8} \times 1.500 \times 40^2 = 300 \text{ ft-kips}$$
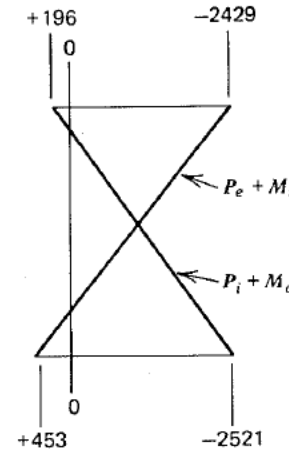
PRESTRESSED CONCRETE- Example

The required section moduli with respect to the top and bottom surfaces of the concrete beam are found from Eqs. (4.1) and (4.2):

$$S_1 \geq \frac{(1-R)M_o + M_d + M_l}{Rf_{ti} - f_{cs}} = \frac{(0.15 \times 50 + 300)12{,}000}{0.85 \times 195 + 2{,}700} = 1{,}288 \text{ in.}^3$$

$$S_2 \geq \frac{(1-R)M_o + M_d + M_l}{f_{ts} - Rf_{ci}} = \frac{(0.15 \times 50 + 300)12{,}000}{465 + 0.85 \times 2{,}520} = 1{,}415 \text{ in.}^3$$

FIGURE 4.3 Beam with variable eccentricity of tendons. (*a*) Cross section dimensions. (*b*) Stresses at midspan.

PRESTRESSED CONCRETE- Example

These values are so nearly the same that a symmetrical beam will be adopted. The 28-in. depth I-section shown in Fig. 4.3a will meet the requirements, and has the following properties:

$$I_c = 19{,}904 \text{ in.}^4 \ (8.28 \times 10^9 \text{ mm}^4)$$

$$S = 1{,}422 \text{ in.}^3 \ (23.3 \times 10^6 \text{ mm}^3)$$

$$A_c = 240 \text{ in.}^2 \ (155 \times 10^3 \text{ mm}^2)$$

$$r^2 = 82.9 \text{ in.}^2$$

$$w_o = 250 \text{ plf (as assumed)}$$

Next, the concrete centroidal stress is found from Eq. (4.4):

$$f_{cci} = f_{ti} - \frac{c_1}{h}(f_{ti} - f_{ci}) = 195 - \frac{1}{2}(195 + 2{,}520) = -1{,}163 \text{ psi}$$

and from Eq. (4.5) the initial prestress force is

$$P_i = A_c f_{cci} = 240 \times 1.163 = 279 \text{ kips (1241 kN)}$$

From Eq. (4.6) the required tendon eccentricity at the maximum moment section of the beam is

$$e = (f_{ti} - f_{cci})\frac{S_1}{P_i} + \frac{M_o}{P_i} = (195 + 1{,}163)\frac{1{,}422}{279{,}000} + \frac{50 \times 12{,}000}{279{,}000}$$

$$= 9.07 \text{ in. (230 mm)}$$

Elsewhere along the span the eccentricity will be reduced in order that the concrete stress limits not be violated.

The required initial prestress force of 279 kips will be provided using tendons consisting of 1/4-in. diameter stress-relieved wires. The minimum tensile strength, according to Table 2.1, is $f_{pu} = 240$ ksi, and for normal prestressing wire, the yield strength may be taken as $f_{py} = 0.85 \times f_{pu} = 204$ ksi. According to the ACI Code (Table 3.2), the permissible stress in the wire immediately after transfer must not exceed $0.82f_{py} = 168$ ksi or $0.74f_{pu} = 178$ ksi. The first criterion controls. The required area of prestressed steel is

$$A_p = \frac{279}{168} = 1.66 \text{ in.}^2 \ (1{,}071 \text{ mm}^2)$$

The cross-sectional area of one 1/4-in. diameter wire is 0.0491 in.^{2}; hence, the number of wires required is

$$\text{Number of wires} = \frac{1.66}{0.0491} = 34$$

PRESTRESSED CONCRETE- Example

Two 17-wire tendons will be used, as shown in Fig. 4.3a.

It is good practice to check the calculations by confirming that stress limits are not exceeded at critical load stages. The top and bottom surface concrete stresses produced, in this case, by the separate loadings are:

$$P_i: \quad f_1 = -\frac{279,000}{240}\left(1 - \frac{9.07 \times 14}{82.9}\right) = +618 \text{ psi}$$

$$f_2 = -\frac{279,000}{240}\left(1 + \frac{9.07 \times 14}{82.9}\right) = -2,943 \text{ psi}$$

$$P_e: \quad f_1 = 0.85 \times 618 = 525 \text{ psi}$$

$$f_2 = 0.85(-2,943) = -2,501 \text{ psi}$$

$$M_o: \quad f_1 = -\frac{50 \times 12,000}{1,422} = -422 \text{ psi}$$

$$f_2 = +422 \text{ psi}$$

$$M_d + M_l: \quad f_1 = -\frac{300 \times 12,000}{1,422} = -2,532 \text{ psi}$$

$$f_2 = +2,532 \text{ psi}$$

PRESTRESSED CONCRETE- Example

Thus, when the initial prestress force of 279 kips is applied and the beam self-weight acts, the top and bottom stresses in the concrete at midspan are, respectively:

$$f_1 = +618 - 422 = +196 \text{ psi}$$

$$f_2 = -2,943 + 422 = -2,521 \text{ psi}$$

When the prestress force has reduced to its effective value of 237 kips and the full service load is applied, the concrete stresses are:

$$f_1 = +525 - 422 - 2532 = -2,429 \text{ psi}$$

$$f_2 = -2501 + 422 + 2532 = +453 \text{ psi}$$

These limiting stress distributions are shown in Fig. 4.3b. Comparison with the specified limit stresses confirms that the design is satisfactory.