**SIMON FRASER UNIVERSITY**
SURREY

ENSC 283: **Center of Pressure and Hydrostatic Force on a submerged body**

Objectives

- To understand the hydrostatic pressure distribution
- To verify the location of center of pressure

Apparatus

The apparatus which is shown in Fig. 1 is designed in a way that only the moment due to hydrostatic pressure distribution on the vertical end of water vessel should be included. The water vessel is designed as a ring segment with constant cross-section. The top and bottom faces are concentric circular arcs centered on the pivot so that the resultant hydrostatic force at every point passes through the pivot axis and does not contribute to the moment.
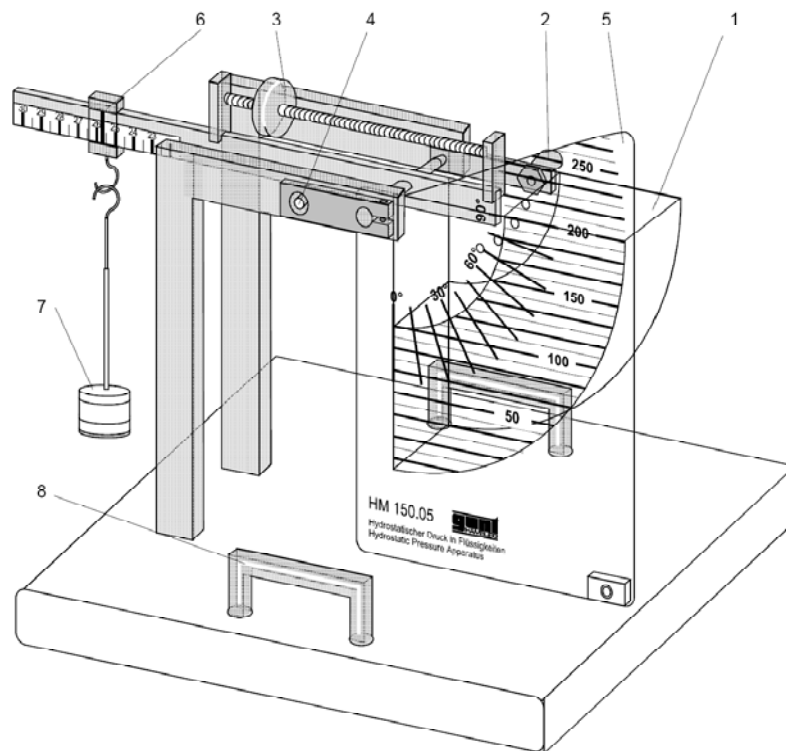
1. Water vessel
2. Detent
3. Slider
4. Stop pin
5. Water level scale
6. Rider
7. Weights
8. Handles

Figure 1- Details of the apparatus.

ENSC 283: **Center of Pressure and Hydrostatic Force on a submerged body**

The radii of the external and internal arcs are 200 and 100 mm, respectively and the width of the vessel is 75 mm.

Theory

The hydrostatic pressure is calculated from:

$$P = \rho g h \tag{1}$$

where ρ is fluid density, g is gravity acceleration and h is the distance from liquid free surface. Because no shear stress exists in a static fluid, all hydrostatic forces on any element of a submerged surface must act in a direction normal to the surface. Equation (1) shows that pressure distribution varies linearly over any inclined surface. The resultant force acting on this surface is related to the pressure on the centroid of the surface, P_c and the surface area, A:

$$F_p = P_c A \tag{2}$$

where P_c itself is related to the depth of center of area of the surface, h_c, through Eq. (1).

As shown in Fig. 2 the resultant force is not therefore applied on the centroid of the surface and the point of action of the resultant force is named as center of pressure. Center of pressure of a plane surface can easily be found using a balance of moments. In Fig. 3 the pressure prism is divided into two sub-volumes and the balance of moments around point O_1 results in:

$$\sum M_{(O_1)} = 0 : A\left(\frac{h}{2} + e\right) = A_1\frac{h}{2} + A_2\frac{2h}{3} \tag{3}$$
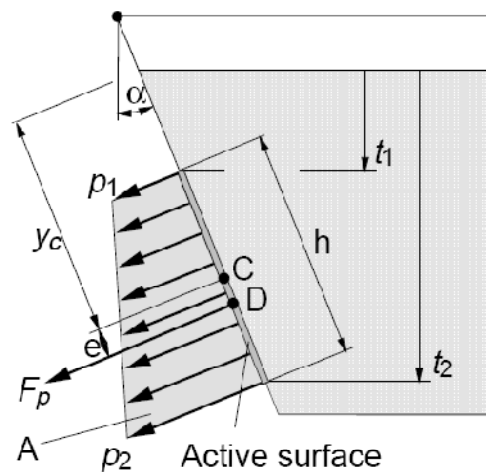

Figure 2- Pressure distribution and resultant force exerted on the plane surface.

ENSC 283: **Center of Pressure and Hydrostatic Force on a submerged body**
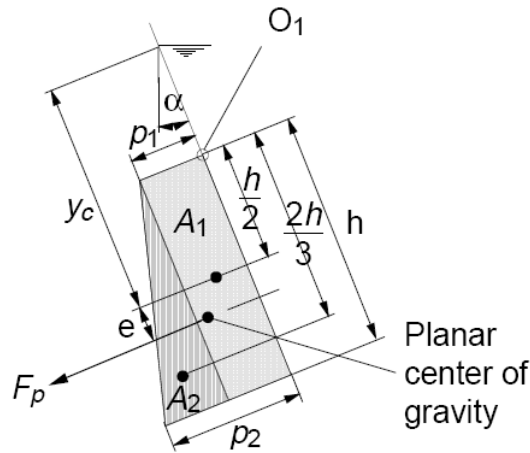

Figure 3- Pressure prism over the plane surface.

where:

$$A_1 = P_1 h \tag{4}$$

$$A_2 = \frac{P_2 - P_1}{2} h \tag{5}$$

$$A = A_1 + A_2 \tag{6}$$

and e is the distance of the center of pressure from the planar centroid of the active surface. Combining Eqns. (3-6) e can be evaluated as:

$$e = \frac{1}{6} h \frac{P_2 - P_1}{P_2 + P_1} = \frac{1}{12} \frac{h^2}{y_c} \tag{7}$$

The water vessel is designed as an arc with constant cross-section. Hence, the force due to weight G of the water always produces the same moment referred to the pinpoint O as the resultant force F_p (see Fig. 4).

$$F_p \cdot y_p = G_t \cdot r \tag{8}$$

Consequently, the apparatus can be used to determine the resultant force and the center of pressure.

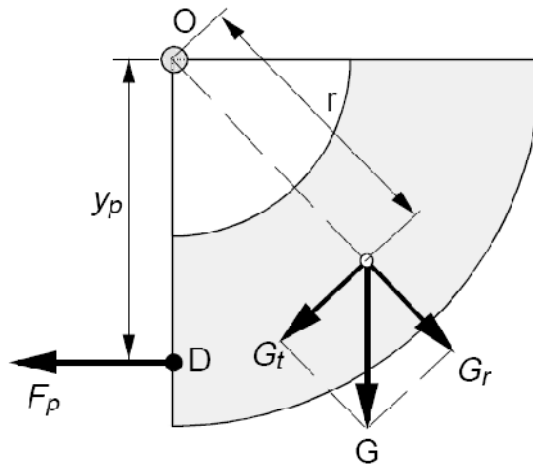ENSC 283: **Center of Pressure and Hydrostatic Force on a submerged body**

Figure 4- Free body diagram of water in the vessel.

At a water level, s, below the 100 mm mark, the height of the active surface changes with water level while for the water level above this mark, the height of the active surface is always 100 mm. Values of distance of center of pressure from surface centroid, e and distance of center of pressure to the pin, l_d, is related to water depth, s, (see Fig. 5):
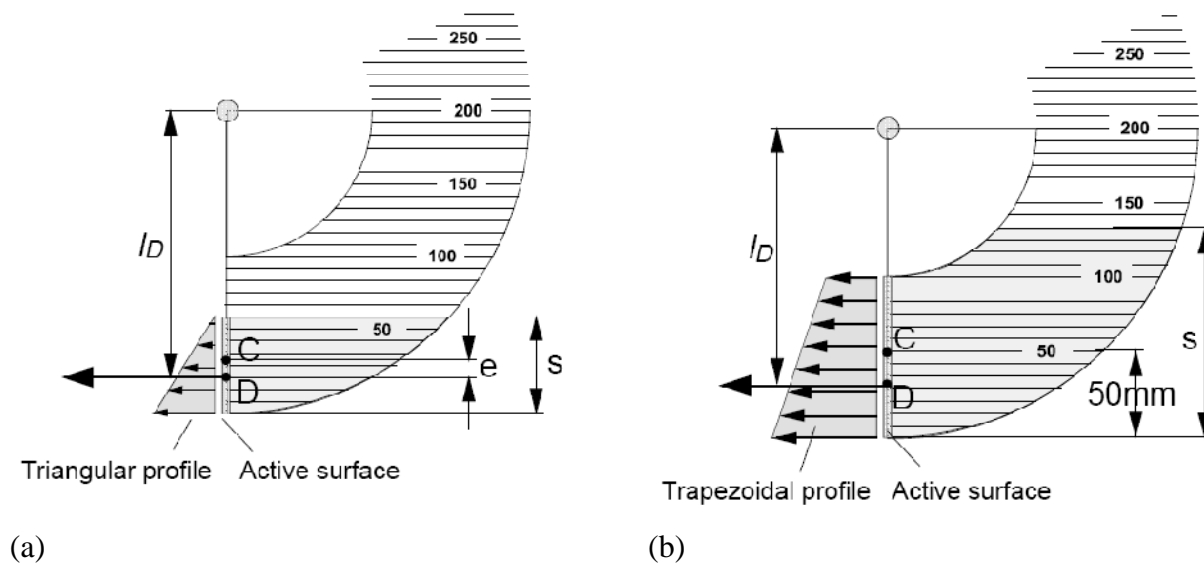

(a)

(b)

Figure 5- Pressure distribution over the effective surface a) $s \leq 100$ b) $s > 100$.

$$e = \frac{s}{6} \ , \qquad l_d = 200mm - \frac{s}{3} \ , \quad s \leq 100 \qquad (9)$$

ENSC 283: **Center of Pressure and Hydrostatic Force on a submerged body**

$$e = \frac{(100\ mm)^2}{12(s - 50\ mm)} \ , \qquad l_d = 150mm + e \ , \quad s > 100 \tag{10}$$

The resultant force is calculated from Eq. (2) where:

$$P_c = \rho g \frac{s}{2} \ , \qquad A_{act} = s \times 75mm \ , \quad s \leq 100 \tag{11}$$

$$P_c = \rho g (s - 50\ mm), \qquad A_{act} = 100mm \times 75mm \ , \quad s > 100 \tag{12}$$

Figure 6 shows how appended weight, F_G, and its lever arm, l, are related to resultant force and its lever arm:
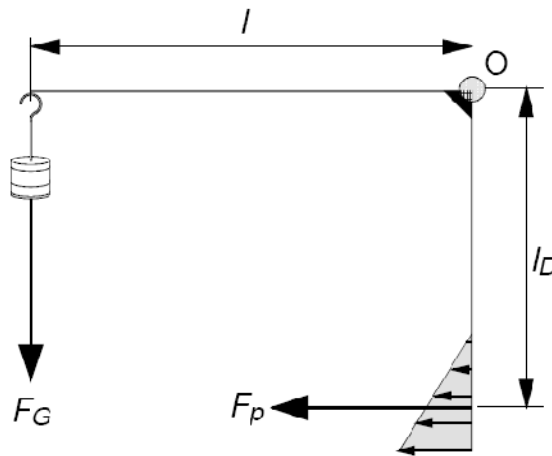
$$F_G . l = F_p . l_D \tag{13}$$

Figure 6- Moment balance around pin point.

This experiment can also be performed with tilted vessel. In this case, if one knows the following parameters:

- s: Water level reading
- α: Tilt angle of the vessel
- s_t: Water level at the lowest point of the vessel (see Fig. 7)
- s_h: Water level of active surface at rim (see Fig. 7)
- h: Height of active surface

can find the center of pressure:

ENSC 283: **Center of Pressure and Hydrostatic Force on a submerged body**

$$h = \frac{s - s_t}{\cos \alpha} \, , \qquad e = \frac{h}{6} \, , \qquad l_d = 200mm - \frac{h}{3} \, , \quad s \le s_h \qquad (14)$$

$$e = \frac{(100 \; mm)^2}{12 \left(\frac{s - s_t}{\cos \alpha} - 50 \; mm \right)} \, , \qquad l_d = 150mm + e \, , \quad s > s_h \qquad (15)$$

The resultant force will be calculated from Eq. (2) using the following values for P_c and A_{act}:

$$P_c = \rho g \frac{s - s_t}{2} \, , \qquad A_{act} = h \times 75mm \, , \quad s \le s_h \qquad (16)$$

$$P_c = \rho g (s - s_t - \cos \alpha . 50 \; mm), \qquad A_{act} = 100mm \times 75mm \, , \quad s > s_h \qquad (17)$$
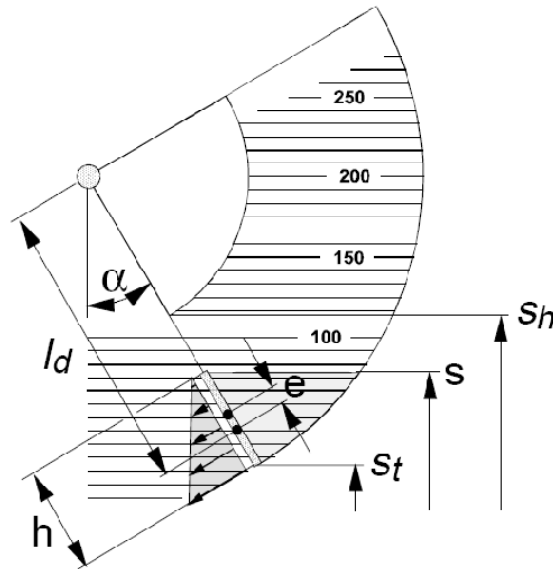

Figure 7- Schematic of the tilted water vessel.

Procedure

- Set the water vessel (1) to the angle of $\alpha = 20°$ using the detent (2) (see Fig. 1).
- Counterbalance the unit with a rotating slider (3). The stop pin (4) must be precisely in the middle of the hole.
- Mount the rider (6), set the lever arm on the scale (e.g. $l = 150mm$).
- Top up with water until the unit is balanced.
- Read off water level, s
- Increase the appended weight in increments of 0.5-1 N and repeat the measurements.

ENSC 283: **Center of Pressure and Hydrostatic Force on a submerged body**

The worksheet in the appendix can be used for reading the required data. Plot the center of pressure and resultant force versus s. Plot s versus appended weight.

Discussion

1) Why is the weight of the vessel and the beam not included in the expression for the center of gravity?

2) How would the location of the center of pressure change if a different fluid were used in the tank?

Appendix

Worksheet for Centre of Pressure

Angle α		Lowest water level , s_t, [mm]	Highest water level, s_h, [mm]
20		15	109
Lever arm, l, [mm]	Appended weight , F_G, [N]	Water level reading, s, [mm]	Calculated lever arm , l_d, [mm]	Resultant force, F_p, [N]