

D.C Motor

Prof. Dr. Suhail Aftab Qureshi

Elect. Engg. Deptt. UET, LHR.

D.C Motor

THE ELECTRIC motor depends for its action on the principle that when a current is passed through a conductor which lies across a magnetic field, it is acted on by a force which tends to move it in a direction perpendicular to itself and to the direction of the field.

Magnitude of the force acting on each conductor = BIl newtons. (See page 89).

Direction of force is given by Fleming's Left-Hand Rule, given in Chapter 10.

D.C Motor

Consider a d.c. machine, field magnets excited and armature supplied from a separate source. All the conductors under one pole will carry current in one direction and under the next pole, of opposite polarity, in the opposite direction (Fig. 88 (a)). By applying the Left-Hand Rule, it will be seen that all the forces acting on the conductors tend to turn the armature in the same direction. The armature will rotate unless the torque exerted by it is less than the resisting torque. (For explanation of torque *see* later.)

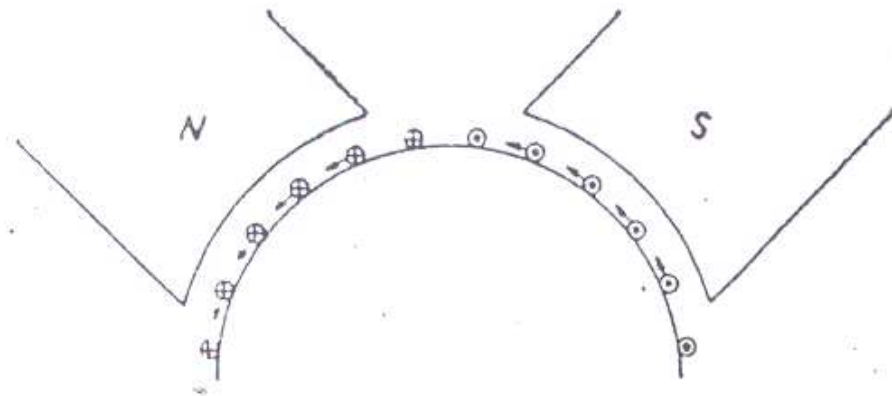


Fig. 88(a). The Motor: Forces acting on Armature Conductors

D.C Motor

Back or Counter E.M.F.

When the armature of a motor rotates, an e.m.f. is induced in the conductors as they cut the lines of force. This e.m.f. is in *opposition* to the applied voltage (V) and is called the *Back or Counter e.m.f.* (E_b); it is the generated e.m.f. produced in the same way as the e.m.f. of a generator.

D.C Motor

The current I_a which flows in the armature is due to the resultant of these two pressures. If $V =$ applied voltage, then

$$I_a = \frac{V - E_b}{R_a}$$

$$\therefore I_a R_a = V - E_b$$

$$E_b = V - I_a R_a \dots\dots\dots(94)$$

Multiply through by I_a . $E_b I_a = V I_a - I_a^2 R_a$.

$V I_a =$ Power supplied to armature (armature input).

$I_a^2 R_a =$ Power wasted in armature (armature copper loss).

$E_b I_a =$ Power available for producing motion (driving power of armature).

E_b determines the armature current and makes the d.c. motor a self-regulating machine.

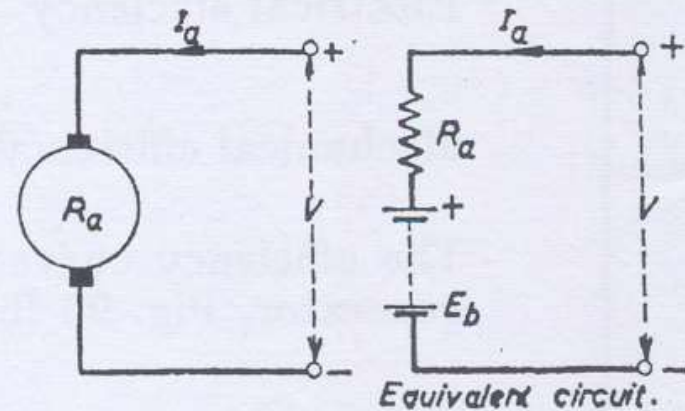


Fig. 88(b).

D.C Motor

Torque

Torque is the term used to express the turning or twisting moment of a force about an axis; it is measured by the product Force \times radius at which force acts.

Consider a wheel or pulley of radius r ft acted on by a circumferential force of F lb (Fig. 89) which causes the wheel to rotate at N rev/min.

Then torque $T = F \times r$ lb ft Work done by the force F in one revolution

$$= \text{Force} \times \text{distance}$$

$$= F \times 2 \pi r \text{ ft lb}$$

Work done by the force F per min.

$$= 2 \pi F r \times N \text{ ft lb}$$

$$= 2 \pi N T \text{ ft lb}$$

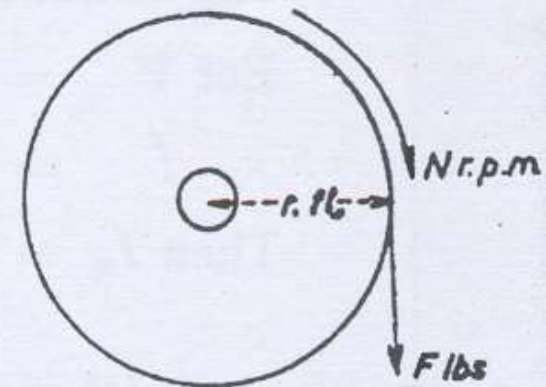


Fig. 89. Torque acting on Pulley

D.C Motor

$$\therefore \text{H.P. developed} = \frac{2 \pi N T}{33000} \dots\dots\dots(95)$$

In a d.c. motor, each conductor lying under a pole face exerts a torque tending to turn the armature; the sum of all these torques is called the *Gross or Armature Torque* (T_a).

D.C Motor

$$\text{Driving power of armature} = E_b I_a \text{ watts} = \frac{E_b I_a}{746} \text{ h.p.}$$

$$\text{Also h.p.} = \frac{2 \pi N T_a}{33000}$$

$$\therefore \frac{2 \pi N T_a}{33000} = \frac{E_b I_a}{746} \text{ and } T_a = \frac{33000}{2 \pi \times 746} \times \frac{E_b I_a}{N}$$

$$T_a = 7.04 \frac{E_b I_a}{N} \dots\dots\dots(96)$$

The whole of this torque is not available for doing useful work, a certain amount being required to overcome the iron and friction losses.

Shaft Torque (T_{sh}) is the torque available for doing useful work, *i.e.*, the actual torque as measured by some form of brake. The horse power obtained using the shaft torque is called the *Brake Horse Power* (b.h.p.).

$$\text{B.H.P.} = \frac{2 \pi N T_{sh}}{33000}$$

$$\therefore T_{sh} = \frac{33000 \times \text{b.h.p.}}{2 \pi N} \dots\dots\dots(97)$$

D.C Motor

$T_a - T_{sh}$ is called the *Lost Torque*; it is equal to

$$7.04 \times \frac{\text{Iron and friction losses}}{N} \dots\dots\dots(98)$$

The back e.m.f. (E_b) may be obtained from:—

(1) the formula $E_b = V - I_a R_a$

(2) the formula $E_b = \frac{2p}{a} \cdot \frac{\Phi N Z}{60}$

(3) the o.c.c.

$$E_b = \frac{2p}{a} \cdot \frac{\Phi N Z}{60} \quad \therefore N = \frac{60 \times a}{Z \times 2p} \times \frac{E_b}{\Phi}$$

$$= K \frac{E_b}{\Phi}, \text{ where } K \text{ is a constant}$$

$$\therefore N = K \frac{V - I_a R_a}{\Phi} \dots\dots\dots(99)$$

Since $N = K \frac{E_b}{\Phi}$, $N \propto \frac{E_b}{\Phi}$ or $E_b \propto N \Phi$

Also $T_a = 7.04 \frac{E_b I_a}{N} = 7.04 \frac{I_a \Phi}{K}$

$$\therefore T_a \propto I_a \Phi \dots\dots\dots(100)$$

D.C Motor

For a given machine,
$$\frac{E_{b1}}{E_{b2}} = \frac{N_1 \Phi_1}{N_2 \Phi_2} \dots\dots\dots(101)$$

$$\frac{T_{a1}}{T_{a2}} = \frac{I_{a1} \Phi_1}{I_{a2} \Phi_2} \dots\dots\dots(102)$$

It can be proved that for a given motor, the maximum driving power is obtained when $E_b = \frac{V}{2}$, but in practice this value of E_b is never approached.

Armature reaction occurs in motors as it does in generators, and all that has been said regarding the latter applies equally well to the former, if the direction of rotation be reversed. (Remember that in the case of a motor, the generated e.m.f. E_b opposes the current.) It will be seen that it is necessary to move the brushes *backwards, i.e.*, to give them *lag* instead of lead. The majority of motors are now fitted with interpoles : the polarity of these must be *the same* as that of the next main pole *astern* of the direction of rotation.

D.C Motor

LOSSES AND EFFICIENCY

The losses which occur in a motor are the same as those which occur in a generator, *viz.*, (1) copper losses (2) iron and friction losses.

The Power Stages

The energy changes and losses which occur in the transformation of electrical energy into mechanical energy can be represented diagrammatically as in Fig. 90 (a).

D.C Motor

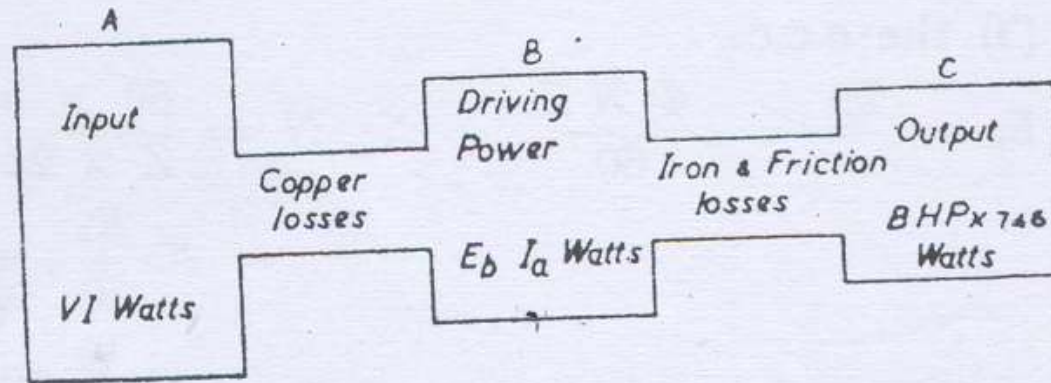


Fig. 90(a). Power Stages for Motor

$A - B =$ Copper losses.

$B - C =$ Iron and friction losses.

Commercial or overall efficiency $\eta_c = \frac{C}{A} \dots\dots\dots(103)$

D.C Motor

Electrical efficiency

$$\eta_e = \frac{B}{A} \dots\dots\dots(104)$$

Mechanical efficiency

$$\eta_m = \frac{C}{B} \dots\dots\dots(105)$$

The efficiency curve of a motor is similar in shape to that of a generator, Fig. 90 (b).

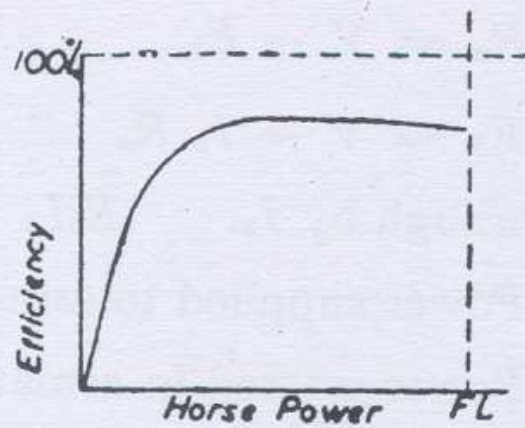


Fig. 90(b). Efficiency Curve of Motor

D.C Motor

MOTOR CHARACTERISTIC CURVES

Like generators, motors are classified according to their method of excitation; they may be of the Shunt, Series or Compound types.

The Shunt Motor

Let V = applied voltage

I = current.

Then $I_a = I - I_{sh}$

$I_{sh} = \frac{V}{R_{sh}}$ is constant (if we neglect

the change in R_{sh} with temperature).

Neglecting armature reaction, Φ may be considered constant at all loads.

Hence, since $T_a \propto I_a \Phi$ (100), we have $T_a \propto I_a$ only; and since $E_b \propto N \Phi$, we have $N \propto E_b$ only.

$$\text{Speed, } N = K \frac{V - I_a R_a}{\Phi}$$

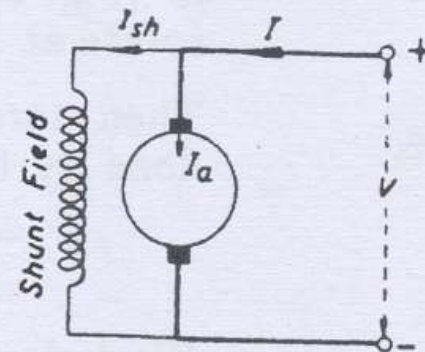


Fig. 91(a). The Shunt Motor

D.C Motor

SPEED CHARACTERISTIC. As the load torque increases, I_a will increase in proportion and therefore the speed will fall slightly, Fig. 91 (b). The smaller R_a , the smaller will be this drop in speed. But due to armature reaction, the flux Φ becomes slightly weaker with increase of load and this may partly or entirely compensate for the effect of armature resistance and the speed of the motor may remain sensibly constant at all loads or it may even increase slightly with increase of load. This latter condition would be unstable: to prevent the speed rising with increase of load a small series winding is added. Generally the speed characteristic is slightly drooping as shown, but the shunt motor is usually regarded as a constant speed machine.

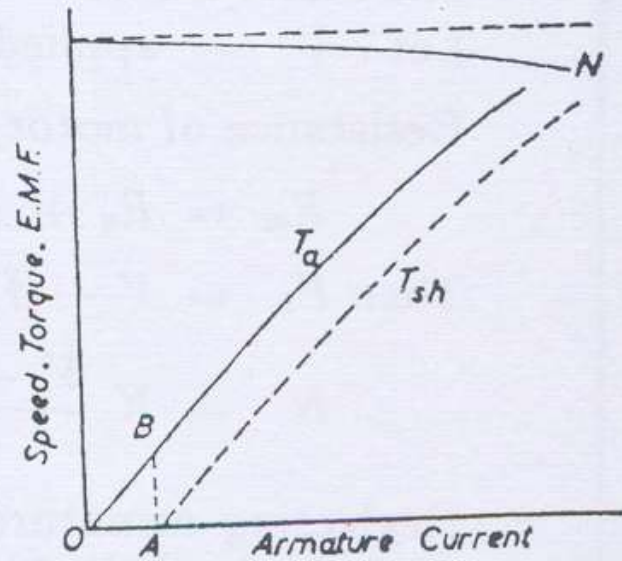


Fig. 91(b). Speed and Torque Characteristics of Shunt Motor

D.C Motor

TORQUE CHARACTERISTIC. If Φ is constant then $T_a \propto I_a$: the torque characteristic is therefore a straight line passing through the origin. Although I_{sh} remains constant, Φ decreases slightly with load due to armature reaction so that the curve bends over as shown. When running light the armature takes a small current OA , the corresponding torque AB being used to overcome the no-load (n.l.) losses, *i.e.*, the iron and friction losses—called the lost torque. The curve of shaft torque T_{sh} is obtained by subtracting this lost torque from the T_a curve.

Starting torque. Assuming a starting current 1.5 times full load current, the torque at starting will be $1.5 \times \text{f.l. torque}$. This is less than for a series motor.

D.C Motor

Example 1. A 250-volt shunt motor takes a total current of 20 amperes. Resistance of shunt field, 200 ohms, and of the armature 0.3 ohms. Find (a) the current in the armature; (b) the back e.m.f.

$$(a) \text{ Current in shunt, } I_{sh} = \frac{250}{200} = 1.25 \text{ amperes}$$

$$\therefore \text{ Current in armature, } I_a = 20 - 1.25 = 18.75 \text{ amperes.}$$

$$(b) \begin{aligned} E_b &= V - I_a R_a \\ &= 250 - 18.75 \times 0.3 \\ &= 244.375 \text{ volts.} \end{aligned}$$

D.C Motor

Example 2. A 220-volt shunt motor takes a total current of 80 amperes and runs at 800 rev/min. Resistance of shunt field, 50 ohms, and of armature, 0.1 ohm. If the iron and friction losses amount to 1,600 watts, find (a) the b.h.p.; (b) the copper losses; (c) the armature torque; (d) the shaft torque; (e) the commercial efficiency.

$$I_{sh} = \frac{220}{50} = 4.4 \text{ amperes}$$

$$\therefore I_a = 80 - 4.4 = 75.6 \text{ amperes}$$

$$E_b = V - I_a R_a$$

$$\therefore E_b = 220 - 75.6 \times 0.1 = 212.44 \text{ volts}$$

$$(A) \text{ Input} = 220 \times 80 = 17600 \text{ W}$$

$$(B) \text{ Driving Power} = 212.4 \times 75.6 = 16050 \text{ W}$$

$$\text{Iron and friction losses} = 1600 \text{ W}$$

$$(C) \text{ Output} = \underline{\underline{14450 \text{ W}}}$$

D.C Motor

$$\begin{aligned} \text{(a) B.H.P.} &= \frac{14450}{746} = 19.37 \\ \text{(b) Copper losses} &= (A - B) = 1550 \text{ W} \\ \text{(c) Armature torque } T_a &= \frac{7.04 E_b I_a}{N} \\ &= \frac{7.04 \times 16050}{800} \text{ lb ft} \\ &= 141.2 \text{ lb ft} \\ \text{(d) Shaft torque } T_{sh} &= \frac{33000 \times 19.37}{2\pi \times 800} \text{ lb ft} \\ &= 127.3 \text{ lb ft} \\ \text{(e) Commercial efficiency, } \eta_c &= \frac{\text{Output}}{\text{Input}} = \frac{14450}{17600} \times \frac{100}{1} \text{ per cent} \\ &= 82.1 \text{ per cent} \end{aligned}$$

D.C Motor

Example 3. A 220-volt shunt motor takes 6 amperes on no load and runs at 750 rev/min. Resistance of shunt field, 110 ohms, and of armature, 0.25 ohm. Calculate the speed when loaded and taking a current of 52 amperes. Assume that armature reaction weakens the field by 4 per cent.

$$I_{sh} = \frac{220}{110} = 2 \text{ amperes.}$$

$$\text{At N.L., } I_a = 6 - 2 = 4 \text{ amperes}$$

$$\begin{aligned} \therefore E_{b1} &= V - I_a R_a \\ &= 220 - 4 \times 0.25 \\ &= 219 \text{ volts.} \end{aligned}$$

D.C Motor

$$\text{When loaded } I_a = 52 - 2 = 50 \text{ amperes}$$

$$\therefore E_{b2} = 220 - 50 \times 0.25 = 207.5 \text{ volts.}$$

$$\text{Now } \frac{E_{b1}}{E_{b2}} = \frac{N_1 \Phi_1}{N_2 \Phi_2}$$

$$\therefore \frac{219}{207.5} = \frac{750 \times \Phi_1}{N_2 \times 0.96 \Phi_1}$$

$$N_2 = \frac{207.5 \times 750}{219 \times 0.96} \\ = 740 \text{ rev/min.}$$

D.C Motor

THE RUNNING-LIGHT TEST

When a shunt motor is running and doing no useful work it is said to be "running light". The power developed by the armature under these circumstances is entirely used up in overcoming the iron and friction losses. These losses, in the case of a shunt motor, are assumed to be constant at all loads; this is not strictly accurate since the iron losses increase slightly with load.

Example 4. A shunt motor takes 5 amperes at 100 volts when running light. Shunt field resistance, 50 ohms; armature resistance, 0.2 ohm. Find the iron and friction losses, the h.p. and efficiency when the motor takes 52 amperes.

D.C Motor

Running light.

$$I_{sh} = \frac{100}{50} = 2 \text{ amperes}$$

$$\therefore I_a = 5 - 2 = 3 \text{ amperes}$$

$$E_b = 100 - 3 \times 0.2 = 99.4 \text{ volts}$$

(B) \therefore Driving power $E_b I_a = 99.4 \times 3 = 298 \text{ W}$

(C) Output is zero.

$$\therefore \text{Iron and friction losses} = 298 \text{ W}$$

When motor takes 52 amperes

$$I_{sh} = 2 \text{ amperes}$$

$$I_a = 52 - 2 = 50 \text{ amperes}$$

$$E_b = 100 - (50 \times 0.2) = 90 \text{ volts.}$$

(A) Input $= 100 \times 52 = 5200 \text{ W}$

(B) Driving Power $= 90 \times 50 = 4500 \text{ W}$

Iron and friction losses $= 298 \text{ W}$

(C) Output $= \underline{\underline{4202 \text{ W}}}$

$$\text{B.H.P.} = \frac{4202}{746} = 5.65 \text{ h.p.}$$

$$\eta_c = \frac{4202}{5200} \times 100 \text{ per cent} = 80.8 \text{ per cent}$$

D.C Motor

The Series Motor

Let V = applied voltage.

Resistance of motor

$$R_m = R_a + R_{se}$$

Then $E_b = V - I_a R_m$ and

$$N = K \frac{V - I_a R_m}{\Phi} \dots\dots\dots(106)$$

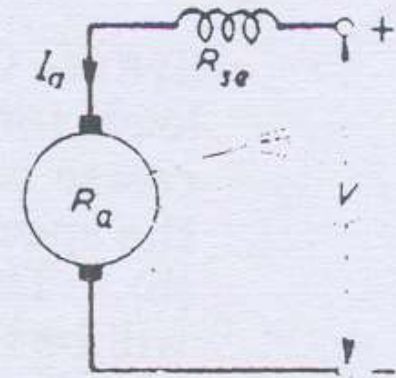


Fig. 92(a). The Series Motor

Neglecting armature reaction, the value of the flux Φ will vary with the load current according to the o.c.c. of the motor: taking armature reaction into account, the useful flux will fall slightly below this.

D.C Motor

SPEED CHARACTERISTIC

At light loads, torque is small; hence I_a and Φ will be small and from above equation, the speed will be high. As the load torque increases, I_a and Φ will increase and N will rapidly decrease. Whilst working on the initial straight portion of the o.c.c., $\Phi \propto I_a$; and if we neglect the small $I_a R_m$ drop, then $N = K \frac{V}{I_a}$, i.e., NI_a is constant.

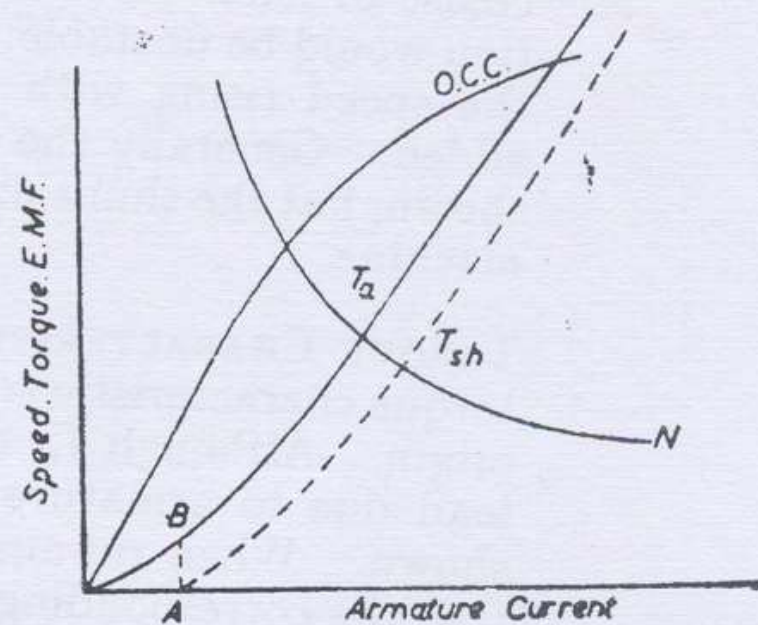


Fig. 92(b). Speed and Torque Characteristics of Series Motor

D.C Motor

Hence the initial portion of the speed curve approximates to a rectangular hyperbola. As the iron approaches saturation, the decrease in N is not so rapid, and further increase in current causes only a small increase in Φ and the curve flattens out. Owing to the rapid increase in speed at light loads, the load on a series motor must never be removed.

TORQUE CHARACTERISTIC

$T_a \propto I_a \Phi$. At light loads, whilst working on the straight portion of the o.c.c., $\Phi \propto I_a$. $\therefore T \propto I_a^2$: hence initial portion of torque characteristic will be a parabola. At heavier loads, when the iron is becoming saturated, the curve merges into a straight line as shown. The T_{sh} curve is obtained as previously by subtracting the lost torque from the T_a curve. In the case of the series motor, owing to the wide variation in speed and flux density,

D.C Motor

the lost torque is not constant, but as a rough approximation it may be assumed to be constant.

Starting Torque. Suppose the starting current is, say 1.5 times f.l. current; further suppose this increases the flux to 1.2 times f.l. then

$$\begin{aligned}\text{Starting torque} &= 1.5 \times 1.2 \times \text{f.l. torque} \\ &= 1.8 \times \text{f.l. torque.}\end{aligned}$$

i.e., 50 per cent increase in current produces 80 per cent increase in torque.

Note that in the case of the series motor, due to the variation of speed and flux/pole, there are *no constant losses*.

Example 5. A 220-volt series motor takes 50 amperes. Armature resistance, 0.1 ohm, series field resistance, 0.08 ohm. If the iron and friction losses are equal to the copper losses at this load, find the b.h.p. and the commercial efficiency.

D.C Motor

$$\begin{aligned} \text{Resistance of motor } R_m &= 0.1 + 0.08 &= 0.18 \text{ ohm} \\ E_b = V - I_a R_m &= 220 - 50 \times 0.18 &= 211 \text{ volts} \\ \text{(A) Input} &= 220 \times 50 &= 11000 \text{ W} \\ \text{(B) Driving Power} &= 211 \times 50 &= 10550 \text{ W} \\ \therefore \text{Copper losses} & &= \underline{450 \text{ W}} \\ \therefore \text{Iron and friction losses} & &= \underline{450 \text{ W}} \\ \text{(C) Output} & &= \underline{10100 \text{ W}} \\ \text{B.H.P.} &= \frac{10100}{746} &= 13.54 \\ \text{Commercial efficiency} &= \frac{10100}{11000} \times \frac{100}{1} &= 91.8 \text{ per cent} \end{aligned}$$

D.C Motor

Example 6. A 110-volt series motor takes 50 amperes when running at 800 rev/min. Armature resistance, 0.25 ohm; series field resistance, 0.15 ohm. Iron and friction losses amount to 800 watts. Find (a) the b.h.p.; (b) the total torque; (c) the shaft torque

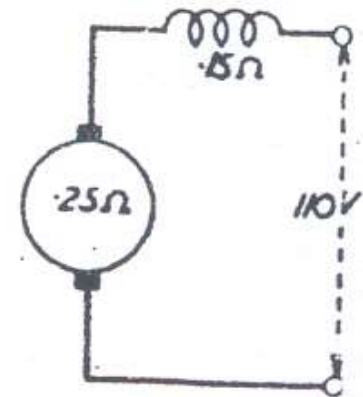
Resistance of motor,

$$R_m = 0.25 + 0.15 = 0.4 \text{ ohm}$$

$$E_b = V - I_a R_m = 110 - 50 \times 0.4 = 90 \text{ volts.}$$

$$\text{(A) Input} = 110 \times 50 = 5500 \text{ W}$$

$$\text{(B) Driving Power} = 90 \times 50 = 4500 \text{ W}$$



D.C Motor

$$\begin{aligned} \text{Iron and friction losses} &= 800 \text{ W} \\ \text{(C) Output} &= 3700 \text{ W} \\ \text{(a) B.H.P.} &= \frac{3700}{746} = 4.961 = 5 \text{ approx.} \\ \text{(b) Total torque} &= 7.04 \frac{E_b I_a}{N} = \frac{7.04 \times 4500}{800} \text{ lb ft} \\ &= 39.6 \text{ lb ft} \\ \text{(c) Shaft torque} &= \frac{33000 \times 4.961}{2 \pi \times 800} = 32.57 \text{ lb ft} \end{aligned}$$

D.C Motor

Example 7. A 250-volt series motor runs at 500 rev/min. The shaft torque is 96 lb ft and the efficiency at this load is 88 per cent. Find the current taken.

Let I amperes be current taken.

$$\text{Input} = 250 \times I \text{ watts.}$$

$$\text{Output} = 0.88 \times 250 \times I \text{ watts.}$$

$$T_{sh} = \frac{33000 \times \text{b.h.p.}}{2 \pi N}$$

$$96 = \frac{33000 \times \text{b.h.p.}}{2 \pi \times 500}$$

$$\therefore \text{B.H.P.} = \frac{96 \times 1000\pi}{33000} = 9.137$$

$$\therefore 0.88 \times 250 \times I = 9.137 \times 746$$

$$I = \frac{9.137 \times 746}{0.88 \times 250} \text{ amperes.}$$
$$= 30.98 \text{ amperes.}$$

D.C Motor

Example 8. A four-pole, 250-volt series motor has a wave-connected armature with 1,254 conductors. The flux per pole is 22 mWb when the motor is taking 50 amperes. Iron and friction losses amount to 1 kW. Armature resistance, 0.2 ohm; series field resistance, 0.2 ohm. Calculate (a) the speed; (b) the b.h.p.; (c) the shaft torque (d) the efficiency at this load.

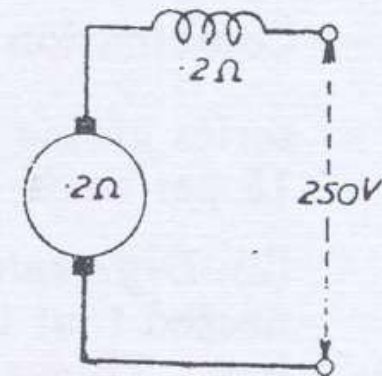
Power

$$\text{input} = 250 \times 50 = 12500 \text{ watts}$$

$$E_b = 250 - 50 \times 0.4 = 230 \text{ volts}$$

$$E_b = \frac{2p}{a} \cdot \frac{\Phi N Z}{60}$$

$$230 = \frac{4}{2} \cdot \frac{22 \times 10^{-3} \times N \times 1254}{60}$$



D.C Motor

$$(a) \therefore N = \frac{230 \times 60 \times 10^3}{44 \times 1254} = 250 \text{ rev/min.}$$

$$\text{Driving Power } E_b I_a = 230 \times 50 = 11500 \text{ W}$$

$$\text{Iron and friction losses} = 1000 \text{ W}$$

$$\therefore \text{Output} = \underline{\underline{10500 \text{ W}}}$$

$$(b) \text{ B.H.P.} = \frac{10500}{746} = 14.08.$$

$$(c) \text{ Shaft torque } T_{sh} = \frac{33000 \times 14.08}{2\pi \times 250} = 295.6 \text{ lb ft}$$

$$(d) \text{ Efficiency } \eta_c = \frac{\text{Output}}{\text{Input}}$$
$$= \frac{10500}{12500} \times \frac{100}{1} \text{ per cent}$$
$$= 84 \text{ per cent.}$$

D.C Motor

Compound Motors

These are of two kinds:—

(1) *Cumulative Compound* in which the series winding is so connected that the field due to it *assists* that due to the shunt winding. The armature torque is therefore increased under heavy load or at starting. As the load increases, the speed decreases in the same way as the series motor, the actual shape of the speed/torque characteristic depending on the degree of compounding. The following types of compound motor with their approved description are used in the Royal Navy.

D.C Motor

Winding of Motor

Shunt motor with series steadying or stabilising turns added to give a 10 per cent drop in speed.

Series motor with limiting shunt turns added.

Combination of about $\frac{2}{3}$ shunt and $\frac{1}{3}$ series giving a drop in speed of 15 per cent—20 per cent.

Approved description

Shunt with stability turns.

Limiting shunt compound.

Cumulative compound.

(2) *Differential Compound* in which the series winding is so connected that the field due to it *opposes* that due to the shunt winding. Thus as the load on the motor is increased, the flux per pole is reduced thereby tending to keep the speed constant. The series winding must be short circuited when starting up, otherwise the

D.C Motor

series field (due to the lower inductance of the series winding) would rise to its full value before the shunt field, with the result that the motor would start in the wrong direction.

A motor of this type must not be overloaded. If the armature current is large, the series field may weaken the shunt field to such an extent that the torque developed would be insufficient to overcome the resisting torque: the motor would stop and take an excessive current. These motors are seldom used, since the shunt motor usually satisfies the requirement of constant speed.

D.C Motor

Example 9. A 220-volt shunt motor when running light takes 7 amperes and runs at 1,250 rev/min. Armature resistance, 0.1 ohm, shunt field resistance, 110 ohms. A compound winding of resistance 0.05 ohm is added, long shunt and cumulatively connected. This winding increases the flux/pole by 20 per cent when the motor is taking its full load current of 62 amperes. Assuming the increase in flux proportional to the armature current and neglecting armature reaction, find the speed (a) when running light; (b) when taking 62 amperes. Compare the torques at 62 amperes with and without the series field.

$$\text{Field current, } I_{sh} = \frac{220}{110} = 2 \text{ amperes}$$

$$\text{When running light, } I_a = 7 - 2 = 5 \text{ amperes}$$

$$E_b = 220 - 5 \times 0.1 = 219.5 \text{ volts}$$

$$N = 1,250 \text{ rev/min.}$$

D.C Motor

60 amperes in series winding gives 20 per cent increase in flux.

1 ampere in series winding gives $\frac{1}{3}$ per cent increase in flux.

(a) When compounded and running light:—

$$\Phi_1 = \Phi + \left(\frac{5}{3} \times \frac{1}{100} \right) \Phi = \frac{61}{60} \Phi$$

$$E_{b1} = 220 - 5 \times 0.15 = 219.25 \text{ volts}$$

$$\frac{E_{b1}}{E_b} = \frac{N_1 \Phi_1}{N \Phi}$$

$$\therefore \frac{219.25}{219.5} = \frac{N_1 \times 61 \Phi}{1250 \Phi \times 60}$$

$$\therefore N_1 = \frac{219.25 \times 1250 \times 60}{219.5 \times 61} = 1228 \text{ rev/min.}$$

D.C Motor

(b) Loaded and taking 62 amperes:—

$$I_a = 60 \text{ amperes}$$

$$\Phi_2 = \Phi + \frac{1}{5} \Phi = \frac{6}{5} \Phi$$

$$E_{b2} = 220 - 60 \times 0.15 = 211 \text{ volts}$$

$$\therefore \frac{211}{219.5} = \frac{N_2 \times 6 \Phi}{1250 \times 5 \Phi}$$

$$N_2 = \frac{211 \times 1250 \times 5}{219.5 \times 6} = 1,000 \text{ rev/min.}$$

$$\frac{\text{Torque with series field}}{\text{Torque without series field}}$$

$$\begin{aligned} &= \frac{60 \times \frac{6}{5} \Phi}{60 \times \Phi} \\ &= \frac{6}{5} \end{aligned}$$

EXERCISE

1. A 100-volt shunt motor is taking a current of 220 amperes. Armature resistance, 0.015 ohm; shunt field resistance, 20 ohms. Calculate the back e.m.f. and the power spent in turning the armature. (96.8 V; 20,807 W)
2. A 220-volt series motor is taking a current of 40 amperes. Resistance of armature, 0.5 ohm; resistance of series field, 0.25 ohm. Calculate:—
 - (a) Voltage at the brushes. (210 V)
 - (b) Back e.m.f. (190 V)
 - (c) Power wasted in armature. (800 W)
 - (d) Power wasted in series field. (400 W)
3. A motor is running at 950 rev/min. and the torque exerted at the pulley is 850 lb ft. What h.p. is being transmitted? (153.8 h.p.)
4. If the power transmitted by the shaft of a motor is 50 h.p. the speed being 480 rev/min. what is the torque? (547.1 lb ft.)

EXERCISE

5. The power transmitted by the shaft of a motor is 20 h.p.: if the torque is 164 lb ft what is the speed? (640 rev/min.)
6. A certain motor takes 350 amperes at 100 volts. If the b.h.p. is 45, what is its efficiency? (95.9 per cent)
7. A shunt motor is taking 240 amperes at 100 volts. Armature resistance, 0.015 ohm; shunt field resistance, 24 ohms. The iron and friction losses amount to 700 watts. What is the b.h.p. output and the mechanical efficiency? (29.56 h.p.; 96.9 per cent)
8. A 200-volt shunt motor is taking a current of 30 amperes. Armature resistance, 0.2 ohm; shunt field resistance, 100 ohms. Iron and friction losses amount to 500 watts. Calculate the b.h.p. and the commercial efficiency. (6.63 h.p.; 82.4 per cent).
9. A shunt motor, armature resistance 0.1 ohm, shunt field resistance 55 ohms, is taking 50 amperes at 220 volts. Iron and friction losses amount to 1,200 watts. Find the back e.m.f. and the b.h.p. of the motor. (215.4 V; 11.66 h.p.)

EXERCISE

10. A shunt motor is taking 84 amperes at 100 volts and developing 9 b.h.p. Armature resistance, 0.05 ohm; shunt field resistance 25 ohms. Find:—

- (a) Copper losses. (720 W)
- (b) Iron and friction losses. (966 W)
- (c) Overall efficiency. (79.9 per cent)

11. A 100-volt shunt motor is developing 6 b.h.p. and the overall efficiency at this load is 86 per cent. Armature resistance, 0.06 ohm; shunt field resistance, 50 ohms. Find:—

- (a) Current taken. (52.05 A)
- (b) Copper losses. (350 W)
- (c) Iron and friction losses. (379 W)

12. A series motor has a total resistance of 0.4 ohm and takes a current of 30 amperes at 100 volts. Iron and friction losses amount to 500 watts. Find:—

- (a) Back e.m.f. (88 V)
- (b) B.H.P. (2.87 h.p.)
- (c) Commercial efficiency. (71.3 per cent)

EXERCISE

13. A 250-volt series motor takes 60 amperes. Armature resistance, 0.08 ohm; series field resistance, 0.06 ohm. If the iron and friction losses are 10 per cent greater than the electrical losses at this load, find the b.h.p. and the commercial efficiency.

(18.7 h.p.; 92.9 per cent)

14. A shunt motor running at 600 rev/min. takes 80 amperes at 250 volts. Armature resistance, 0.1 ohm; shunt field resistance, 50 ohms. Iron and friction losses, 2,188 watts. Find:—

- | | |
|----------------------------|----------------|
| (a) B.H.P. | (21.45 h.p.) |
| (b) Total torque. | (213.4 lb ft.) |
| (c) Shaft torque. | (187.7 lb ft.) |
| (d) Copper losses. | (1,812 W) |
| (e) Commercial efficiency. | (80 per cent) |

EXERCISE

15. A 220-volt shunt motor takes 105 amperes. Armature resistance, 0.08 ohm; shunt field resistance, 44 ohms. The motor runs at 950 r.p.m. If the iron and friction losses are equal to 2 kW, find:—

- (a) B.H.P. (25.74 h.p.)
- (b) Total torque. (157.1 lb ft.)
- (c) Shaft torque. (142.3 lb ft.)

16. The output of a certain shunt motor is 5 b.h.p. when taking 50 amperes at 100 volts and running at 750 rev/min. Armature resistance, 0.06 ohm; shunt field resistance, 50 ohms. Find:—

- (a) Iron and friction losses. (932 W)
- (b) Armature torque. (43.76 lb ft.)
- (c) Shaft torque. (35 lb ft.)

EXERCISE

17. A 220-volt series motor takes a current of 35 amperes and runs at 500 rev/min. Armature resistance, 0.25 ohm; series field resistance, 0.3 ohm. If iron and friction losses amount to 600 watts, find:—

- (a) Armature torque. (98.93 lb ft.)
- (b) Shaft torque. (90.46 lb ft.)
- (c) Overall efficiency. (83.4 per cent)

18. A series motor takes 40 amperes at 220 volts and runs at 800 rev/min. Armature resistance, 0.2 ohm; series field resistance, 0.1 ohm. Iron and friction losses $\frac{1}{2}$ kW. Calculate:—

- (a) Total torque. (73.21 lb ft.)
- (b) Shaft torque. (68.82 lb ft.)
- (c) B.H.P. (10.48 h.p.)

EXERCISE

19. A series motor takes 45 amperes at 210 volts and runs at 750 rev/min. The horse-power output is 10; armature resistance, 0.2 ohm; series field resistance, 0.3 ohm. Find:—

- (a) Iron and friction losses.
- (b) The lost torque.

(977 W)
(9.17 lb ft.)

20. The torque measured at the pulley of a series motor is 96 lb. ft. The motor is taking 50 amperes at 200 volts and runs at 600 rev/min. Armature resistance, 0.3 ohm; series field resistance, 0.2 ohm. Find:—

- (a) B.H.P.
- (b) Iron and friction losses.
- (c) Commercial efficiency.

(10.96 h.p.)
(571 W)
(81.79 per cent)

EXERCISE

21. A 200-volt shunt motor—armature resistance, 0.2 ohm; shunt field resistance, 100 ohms—takes a current of 7 amperes when running light. Find the iron and friction losses. Assuming these losses to remain constant, find the b.h.p. when taking a current of 52 amperes. (995 W; 11.4 h.p.)

22. A shunt motor takes 8 amperes at 220 volts when running light. Armature resistance, 0.3 ohm; shunt field resistance, 88 ohms. Find the b.h.p. and efficiency when the motor is taking 52.5 amperes. (12.12 h.p.; 78.27 per cent)

23. A shunt motor takes 6.5 amperes at 120 volts when running light. Armature resistance, 0.5 ohm; shunt field resistance, 80 ohms. Find the b.h.p. and shaft torque when the motor is taking 66.5 amperes, and running at 800 rev/min. (6.82 h.p.; 44.77 lb ft.)

EXERCISE

24. A four-pole, 440-volt shunt motor has a wave-wound armature with 762 conductors. The full load armature current is 40 amperes and the effective armature resistance is 0.25 ohm. The flux per pole is 25 mWb. Allowing a brush contact drop of 2 volts, calculate the full load speed. (674 rev/min.)

25. A 240-volt shunt motor takes 5 amperes on no load. Armature resistance, 0.25 ohm; field resistance, 200 ohms. Calculate its efficiency when taking full load current of 50 amperes and the percentage change in speed between no load and full load. (85 per cent; 4.7 per cent)

EXERCISE

26. A four-pole, 440 volt, 80-h.p. motor has a full load efficiency of 90 per cent: the armature is wave wound with 470 conductors. Total resistance of armature and interpoles, 0.1 ohms; shunt field resistance, 220 ohms. The flux per pole is 40 mWb. Calculate for full load:—

- (a) Speed. (678 rev/min.)
(b) Useful torque in lb. ft. (619.3 lb ft.)

27. A 220-volt, four-pole series motor has a lap-wound armature, 250 slots, 4 conductors per slot. The gap flux per pole is 20 mWb when the current is 42 amperes. Armature resistance, 0.4 ohm; field resistance, 0.2 ohm. Iron and friction losses, 800 watts. Calculate the pull in lb. at the rim of the motor pulley which is 12 in. in diameter. (177.9 lb.)

28. A shunt generator gives 100 amperes at a terminal p.d. of 100 volts, the speed being 1,200 rev/min. Armature resistance, 0.035 ohm; field resistance, 50 ohms. What will be the speed of the machine when run as a motor taking 102 amperes at 100 volts. (1,118 rev/min.)