HYDROSTATIC FORCES ON SURFACES

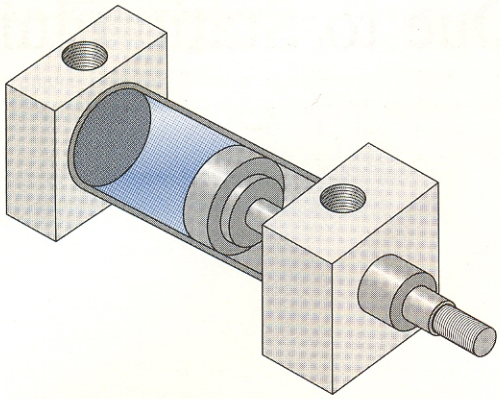
Remember the second law of Pascal –

In a container, pressure acts perpendicular to the boundary

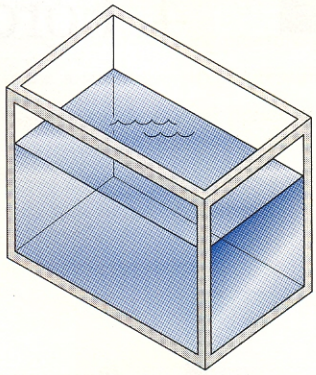
In this lecture we will investigate how forces act on surfaces –

- Nature of plane or curved surface
- Total force
- Center of force

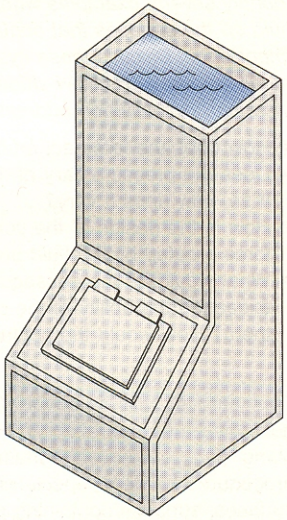
Remember – F = P x A



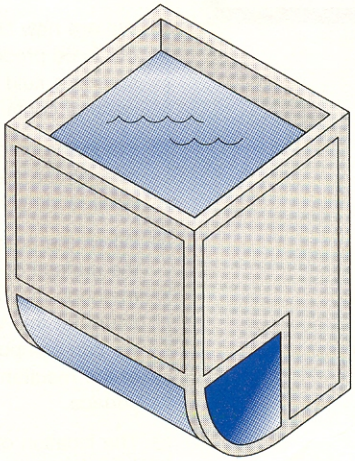
a) Fluid power cylinder



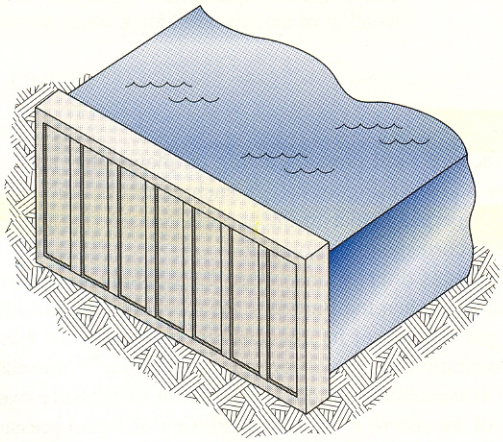
b) Storage tank



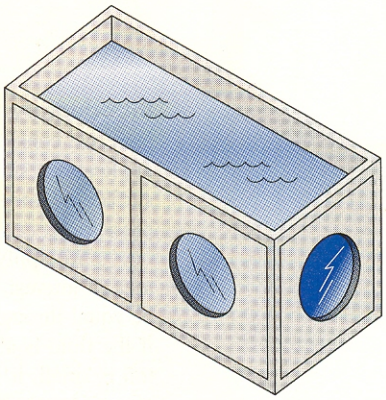
c) Fluid reservoir and hatch



d) Tank with a curved surface



e) Retaining wall



f) Aquarium observation windows

FIGURE 4.1 Examples of cases where forces on submerged areas must be computed.

Example Problem 4.2:

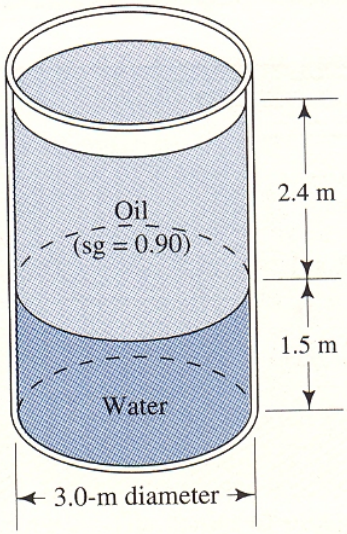


FIGURE 4.3 Cylindrical drum for Example Problem 4.2.

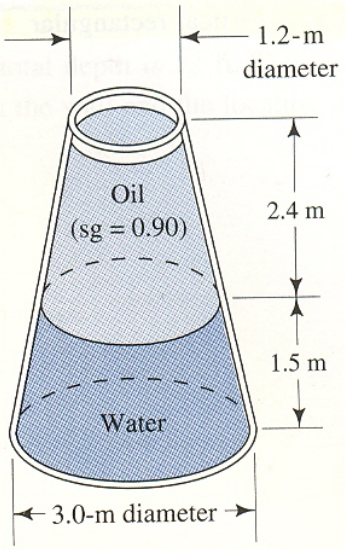


FIGURE 4.4 Cone-shaped container for Example Problem 4.3.

What is the pressure and force at the bottom of the containers??

$P = 0 + \gamma o * 2.4 + \gamma w * 1.5$

$\gamma o = 9.81 * 0.90 = 8.83$ kN/m3

$P = 0 + 8.83 * 2.4 + 9.81 * 1.5$

$P = 0 + 21.2 + 14.7 =$ **35.9 kPa (gage)**

$F = P*A = 35.9*A$
$= 35.9 * \pi (3.0)^2/4$
$=$ **253.8 kN**

Force and pressure for container 2 will be the same! – Pascal's paradox!

Force and pressure are being felt by the insides of the container.

However the weight of the two containers will be different

$W1 = \pi (3.0)^2/4 * [2.4*8.83 + 9.81*1.5]$

= 253.8 kN

Container 2

Volume of frustum of cone =

$V = (\pi/12) * h * (D^2 + Dd + d^2)$

D1 = 1.2
D2 = 2.307
D3 = 3.0

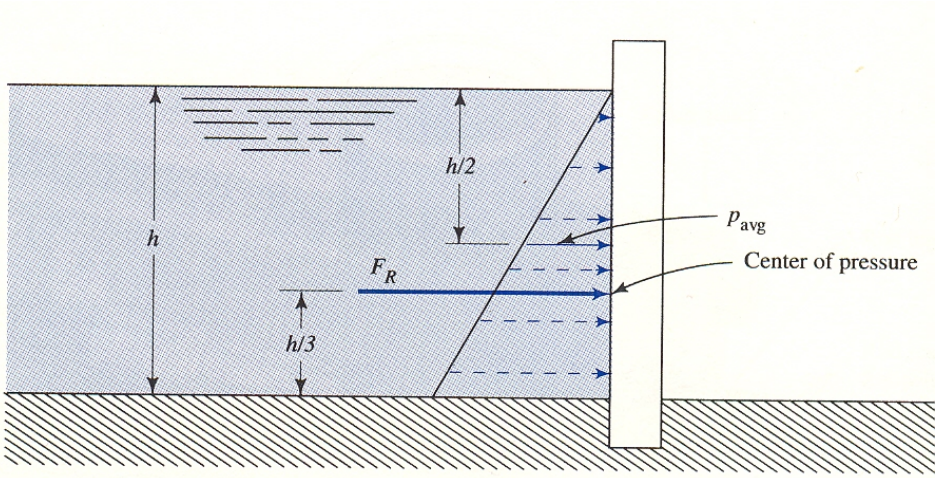
W2 = V1 * 8.83 + V2 * 9.81

$= (\pi/12) * 2.4 * (2.307^2 + 2.307*1.2 + 1.2^2)* 8.83 + (\pi/12) * 1.5 * (3^2 + 3*2.307 + 2.307^2)$

= 134.7 kN

Forces on rectangular walls

FIGURE 4.6 Vertical rectangular wall.



Hoover dam! Water depth ~ 720 ft. Pressure at bottom ??????

Problem 4.4

Determine the force and center of force on a WALL if -

Fluid = gasoline = sg = 0.68

Total depth = 12 ft
Length of wall = 40 ft

Average pressure = γ * h/2

= 0.68 * 62.4 * 12/2 = 254.6

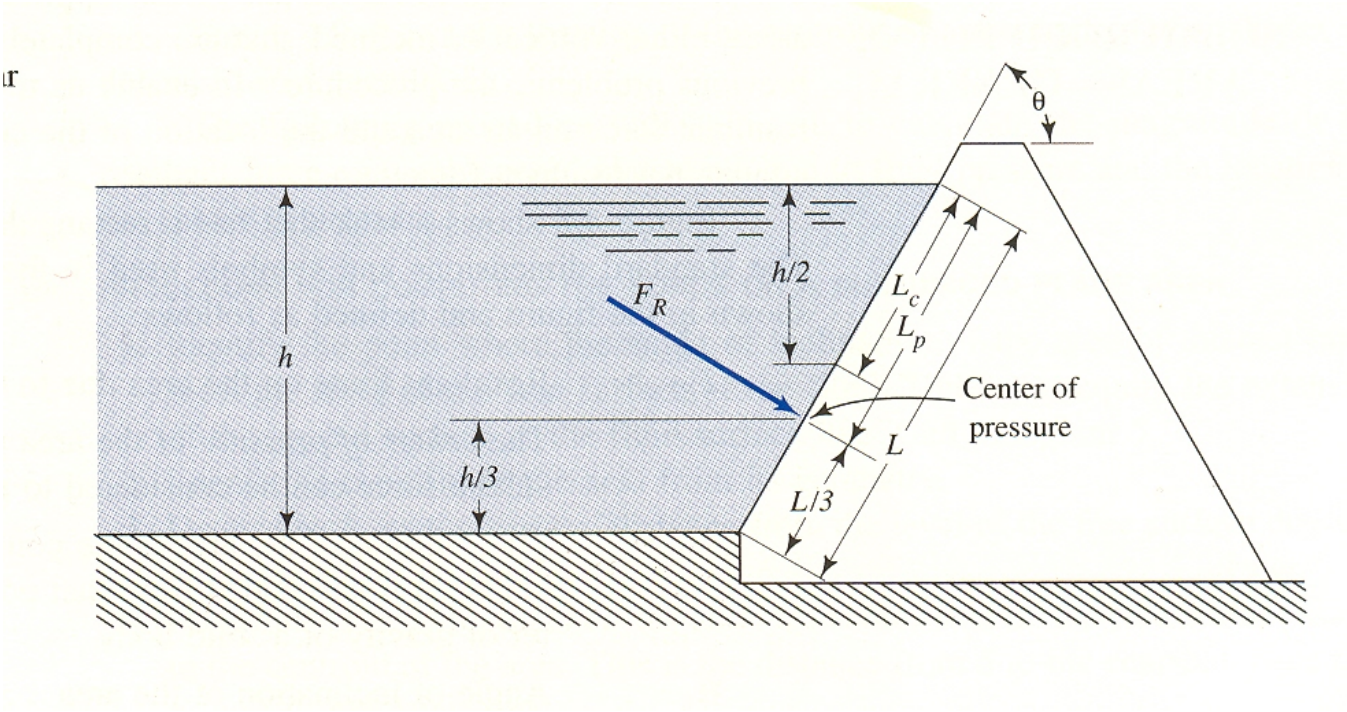
Force on wall = F = p*A
= 254.6 * 12*40 = 122,204 lb

Center of pressure = 12/3 from bottom

= **4 ft from bottom**

Problem 4.5

Dam retaining water.
Dam length = 30.5 m
Depth = 8m
Dam wall inclined at 60 degrees
Calculate force and location of force?



Average pressure = γ * h/2

= 9.81 * 8/2 = 39.24

Length along which pressure acts =

= L = h/sin θ

$= 8 / \sin 60^0$

$= 9.24$ m

Force = pA = 39.24 * 9.24 * 30.5

= 11060 kN

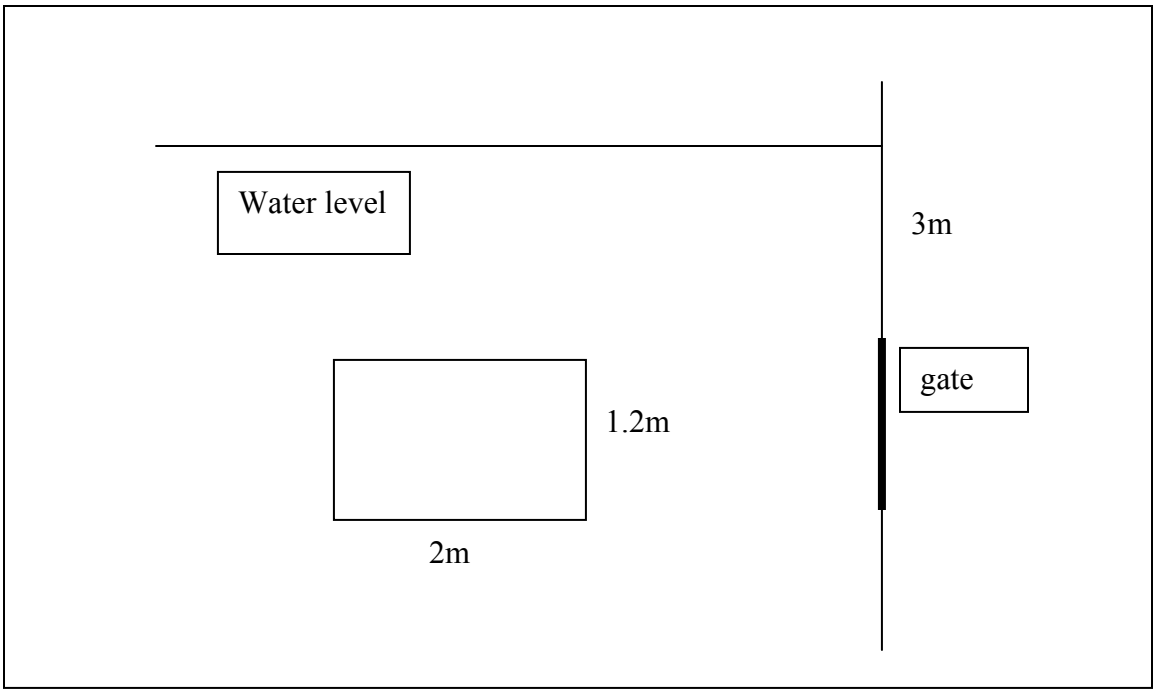
Center of pressure =

h/3 from bottom = 8/3 = **2.67 m**

or

L/3 along inclined plane = 9.24/3 = **3.08 m**

Force on a submerged plane area



Find the force on the gate and the center of pressure.

Steps –

1. find the centroid of the area
2. find the distance from the top of fluids to centroid = hc
3. determine the pressure at the centroid
4. determine the force at the centroid
5. calculate the moment of inertia of the area
6. compute the center of pressure

centroid of given rectangle = 1.2/2 = 0.6 m

hc = 3.0 + 0.6 = 3.6 m

pressure = p = γ * hc

= p = 9.81*3.6 = **35.32 kN/m^2**

Force = pA

= 35.32 * 1.2 * 2 = **84.7 kN**

Now,

Moment of inertia of rectangle = Ic = $BH^3/12$

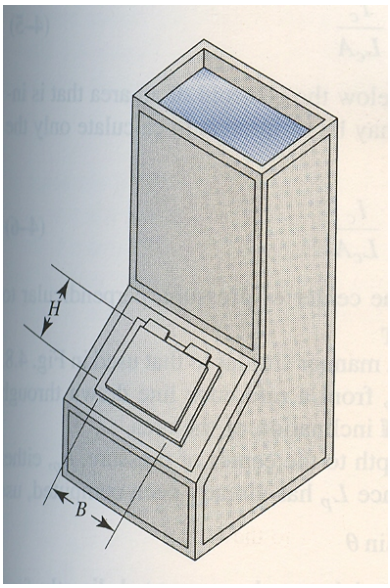
= 2 * $1.2^3/12$ = 0.288

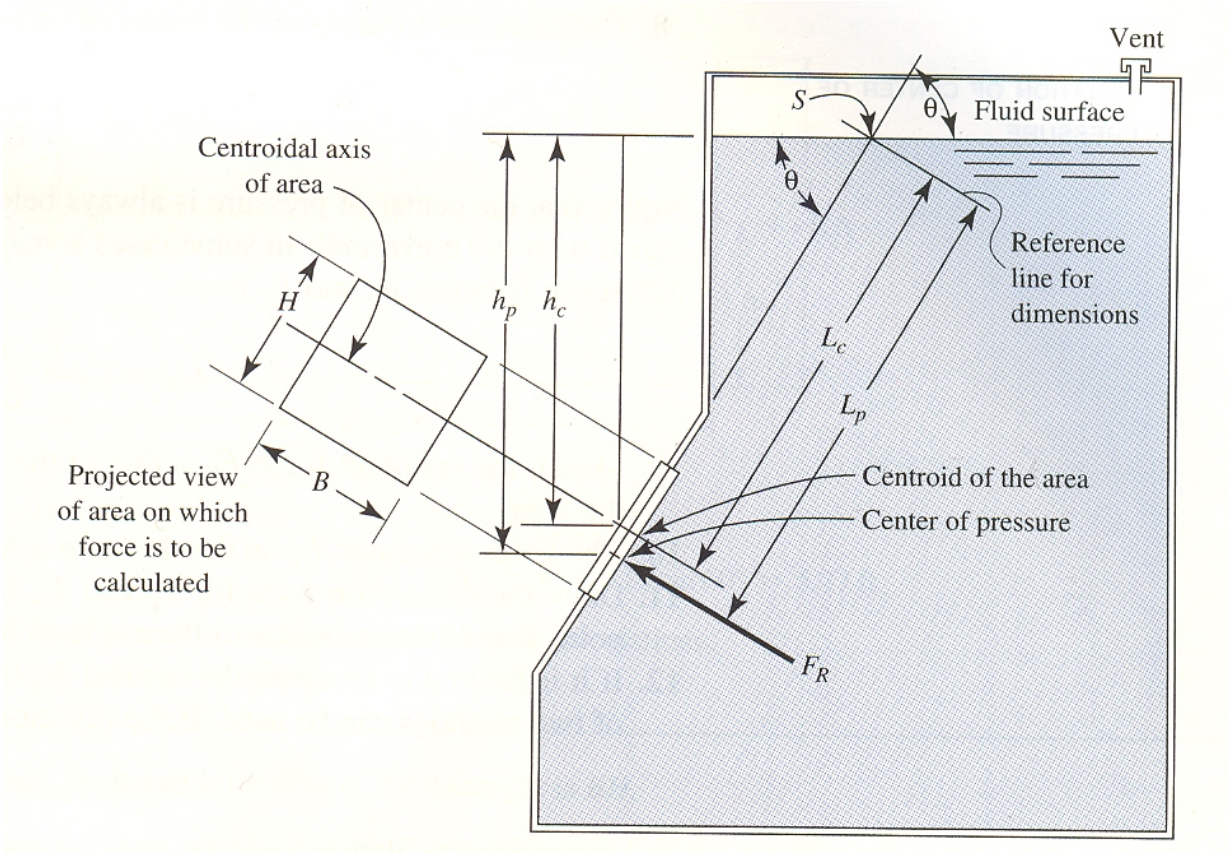
The center of pressure =

= **hc + Ic/(hc*A)**

= 3.6 + 0.288/(3.6*1.2*2) = **3.633 m**

7. Force on a submerged plane area





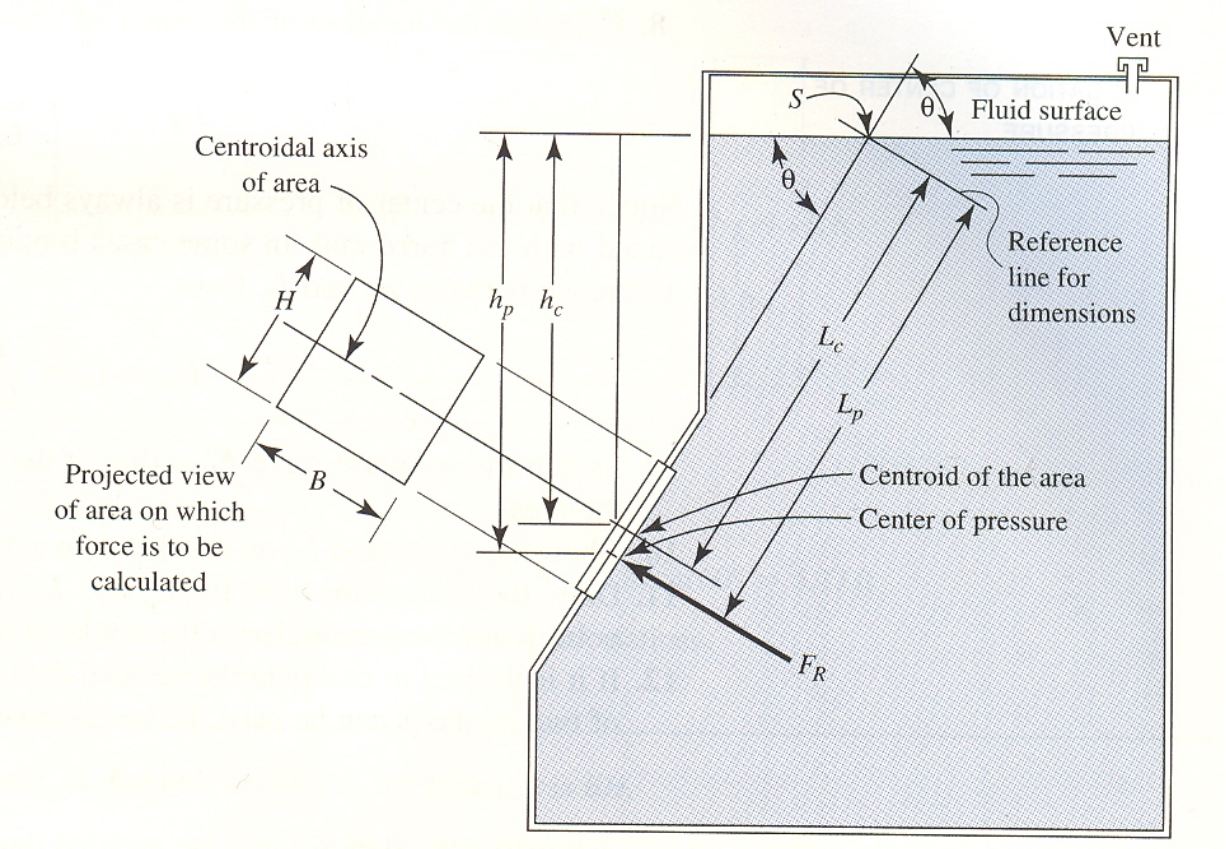
Problem 4.6:

STEPS –

1. determine the point where the angle of inclination intersects the fluid surface
2. locate the centroid of the surface
3. determine hc – vertical distance from fluid surface to centroid
4. determine Lc – inclined distance to centroid

   $hc = Lc \sin\theta$

5. calculate area A
6. calculate force on area = $F_R = \gamma hc * A$
7. calculate the moment of inertia = Ic
8. calculate the center of pressure = $Lp = Lc + Ic/(Lc*A)$
9. Sketch F_R acting on the area
10. $hp = Lp \sin\theta$
11. or $hp = hc + Ic \sin^2\theta/(hc*A)$



GIVEN –
Tank with oil, sg = 0.91

Rectangular gate B = 4ft, H = 2ft

Inclined wall of tank = θ = 60 degrees

Centroid of the gate is = hc = 5ft from the surface

<u>Calculate</u> –

- The force on the gate
- The center of pressure

1. draw a sketch of the system
2. identify the centroid of the gate based on its geometry

3. $h_c = 5ft$

4. determine L_c

$h_c/L_c = \sin \theta$

$L_c = h_c/ \sin \theta = 5/ \sin 60^o = 5.77$ ft

5. area of gate = $A = BH = 4 * 2 = 8ft^2$

6. Determine force on the gate

Fr = $\gamma_o h_c A = 0.91 * 62.4 * 5 * 8 =$ **2270 lb**

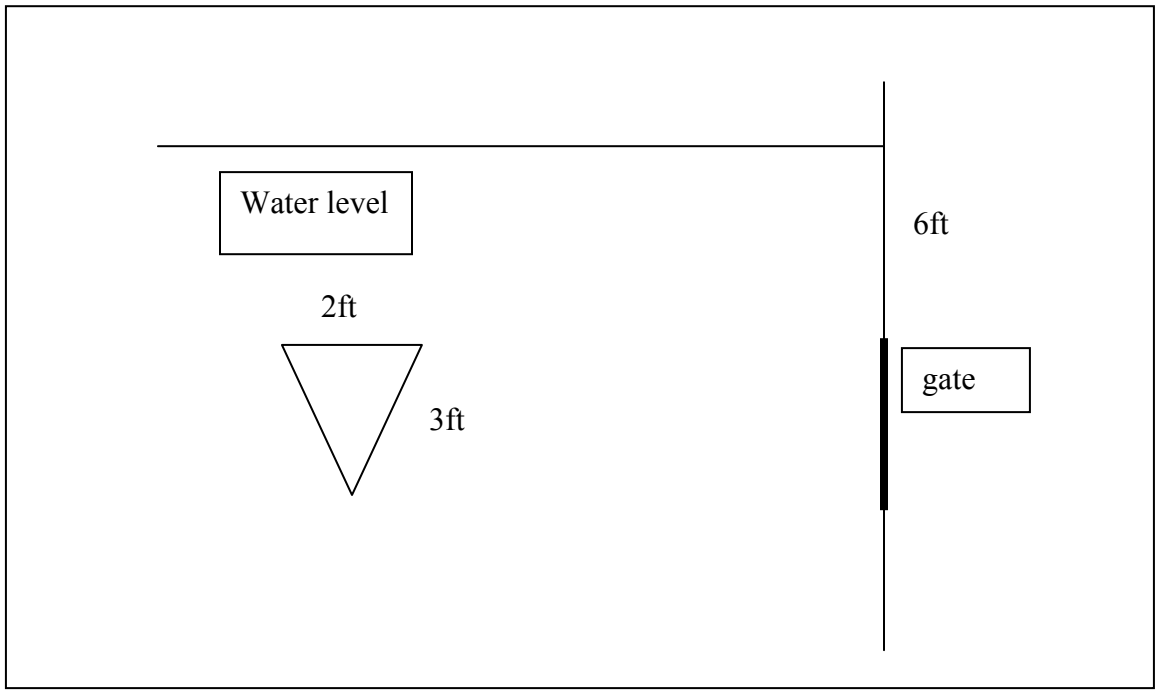
7. Determine center of pressure

moment of inertia of gate = $I_c = BH^3/12 = 4 * 2^3 / 12 = 2.67$ ft^4

Center of pressure = $L_p = L_c + (I_c/L_c*A)$

$= 5.77 + (2.67/5.77 * 8) = 5.77 + 0.058 =$ **5.828 ft**

Example



Centroid at ???????????

Force = $\gamma\ h_c\ A$ = 62.4 * (6+1) * 0.5*2*3

= **1310 lb**

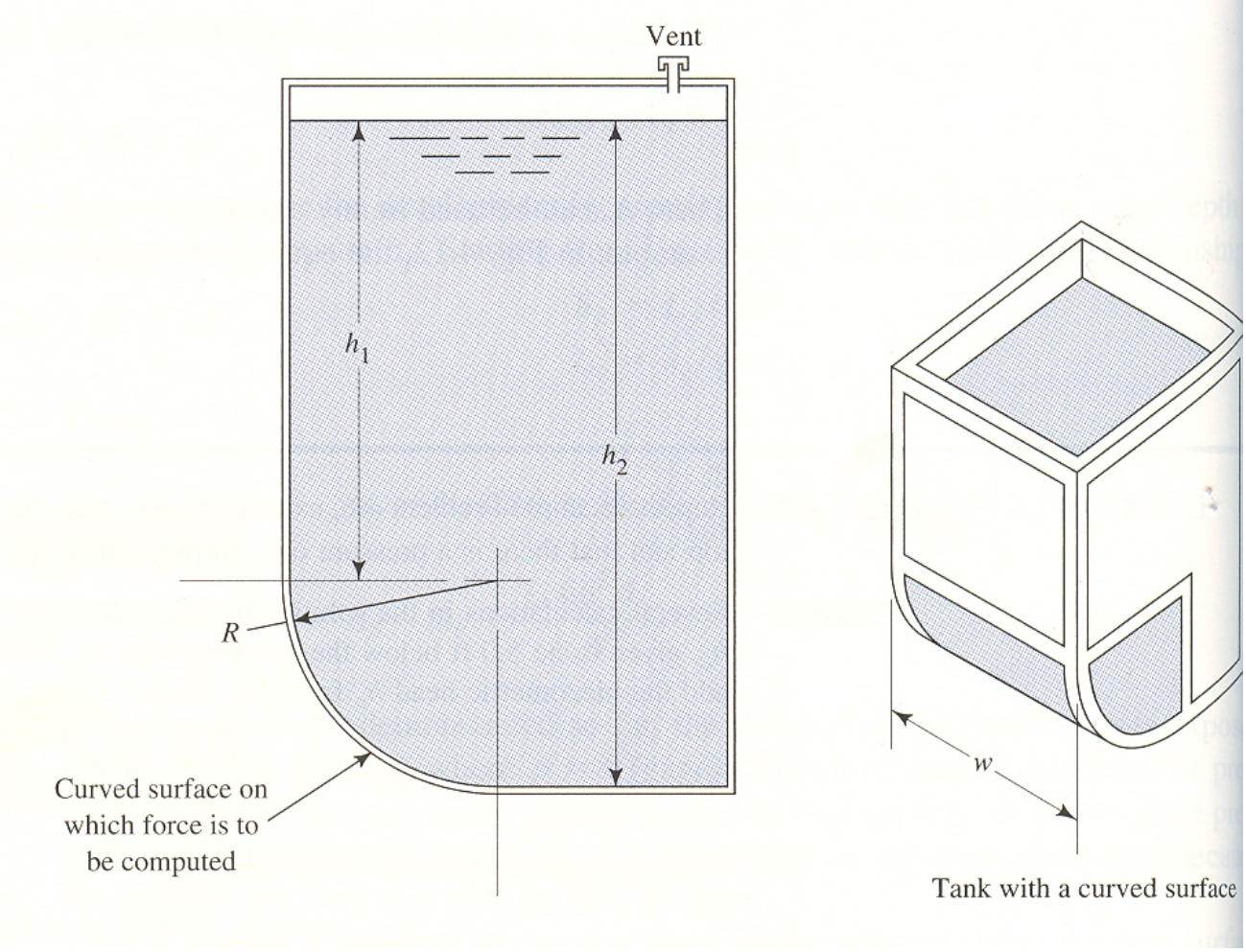
Center of pressure = hp = hc + Ic/(hc*A)

= 7 + [2*3^3/36/(7*0.5*2*3)] = **7.07 ft**

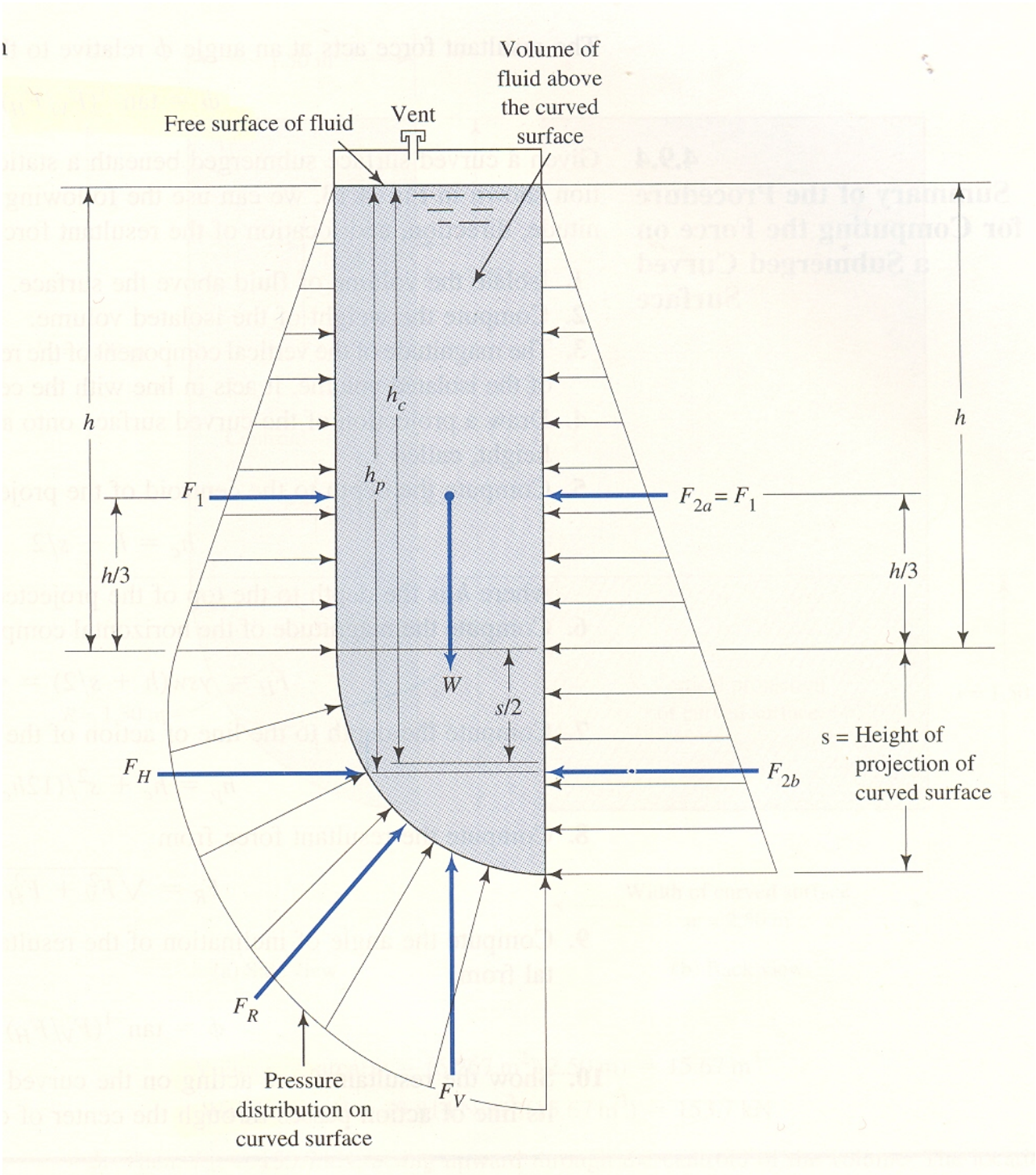
Assignment # 3:

- 4.9E
- 4.11M
- 4.17M
- 4.20M

Distribution of Force on a Curved surface



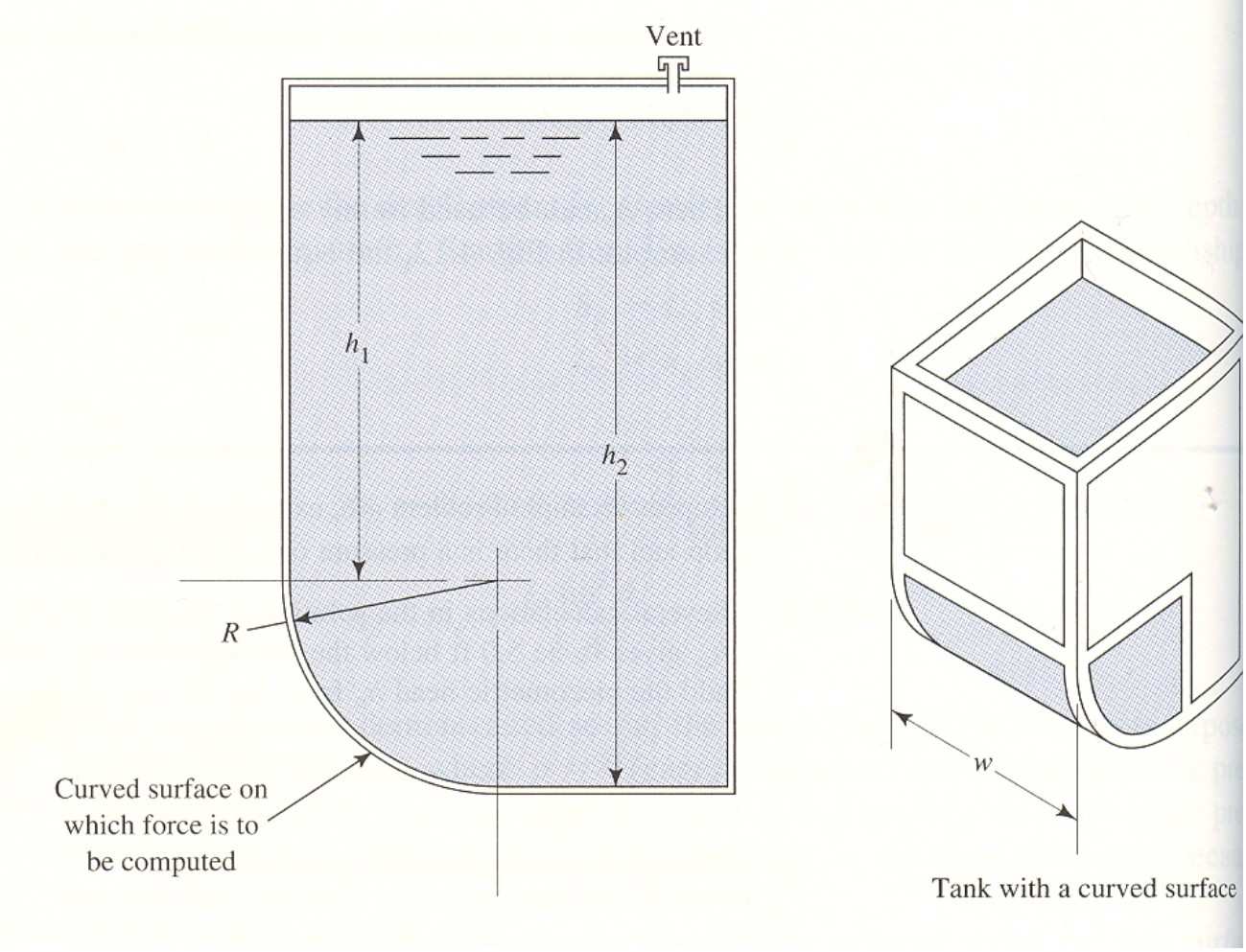
First step – isolate the portion of interest and visualize the forces


Volume of fluid above the curved surface
Free surface of fluid
Vent
h
h_c
h_p
F_1
h/3
W
s/2
F_H
F_R
Pressure distribution on curved surface
F_V
$F_{2a} = F_1$
h
h/3
F_{2b}
s = Height of projection of curved surface

Problem 4.8:

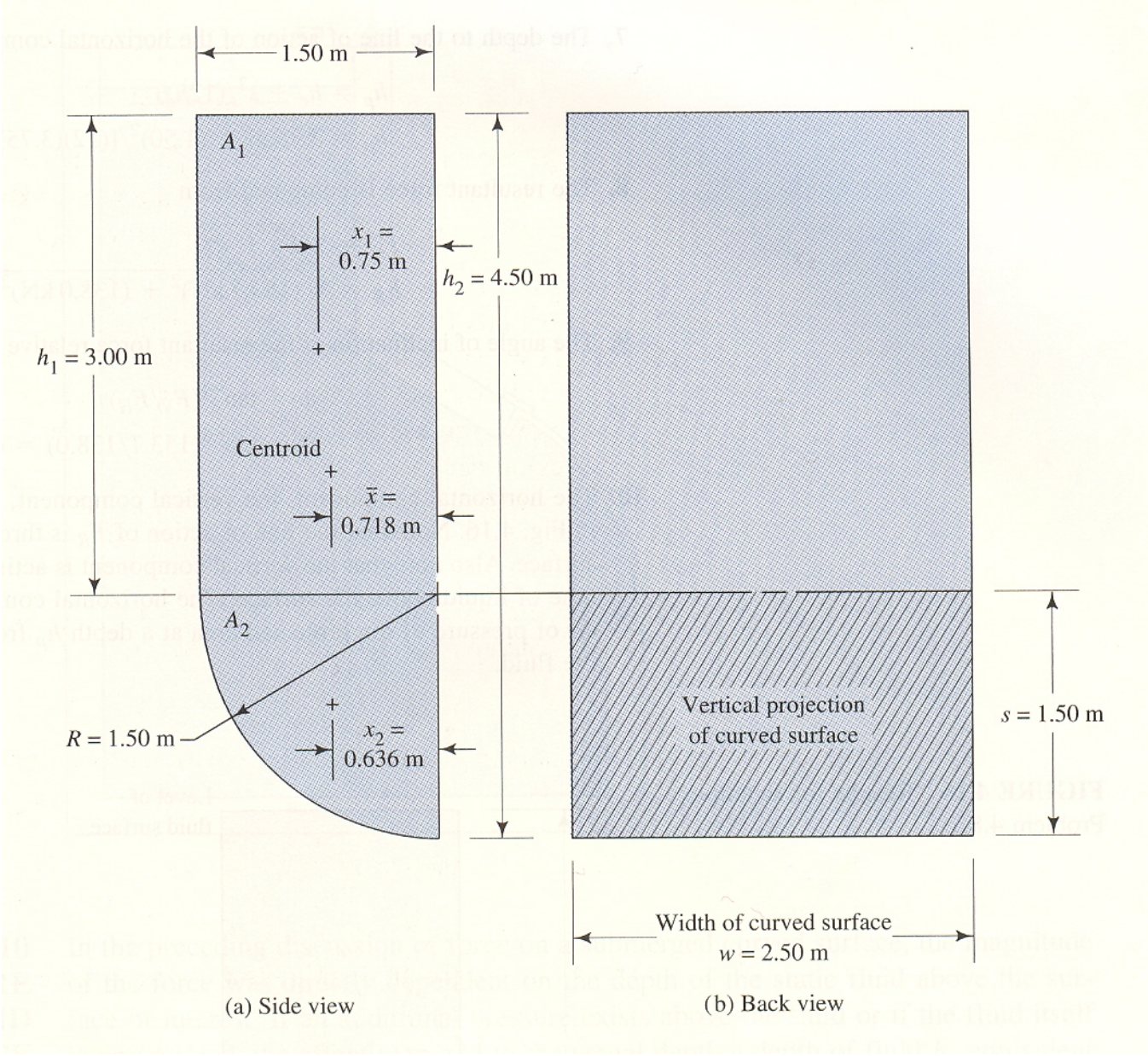
h1 = 3.00 m
h2 = 4.5 m
w = 2.5 m

liquid = water



Compute the **horizontal and vertical components <u>on the curved surface</u> and the resultant force**.

1. the isolated volume is shown below –



Determine the volume above the curved surface

Areas –

Area = A1 + A2 = (3 * 1.5) + ¼ (π *1.5^2)

$= 4.5\ m^2 + 1.767\ m^2 = 6.267\ m^2$

Volume = area x width = 6.267 * 2.5 = 15.67 m^3

Therefore weight = **Fv** = 15.67 * 9.81 = **153.7 kN**

Determine the centroid of the vertical force

For A1, x1 = 0.75

For A2, x2 = 0.424 R (formula for centroid of quadrant)

= 0.424 * 1.5 = 0.636 m

Determine the centroid of the area A1 and A2 =

x = (A1x1+A2x2)/(A1+A2) = 0.718 m

Now let's compute the horizontal force

Depth to the centroid of projected area

h_c = h1 + s/2 = 3.0 + 0.75 = 3.75 m

Magnitude of the horizontal force =

Fh = $\gamma\ h_c$ * sw = 9.81* 3.75*1.5*2.5 = **138.0 kN**

Depth to the line of action of the horizontal force

$hp = h_c + s^2/12\ h_c = 3.75 + 1.5^2 / (12*3.75) = 3.80$ m

Resultant Force = $Fr = (Fv^2+Fh^2)^{1/2}$

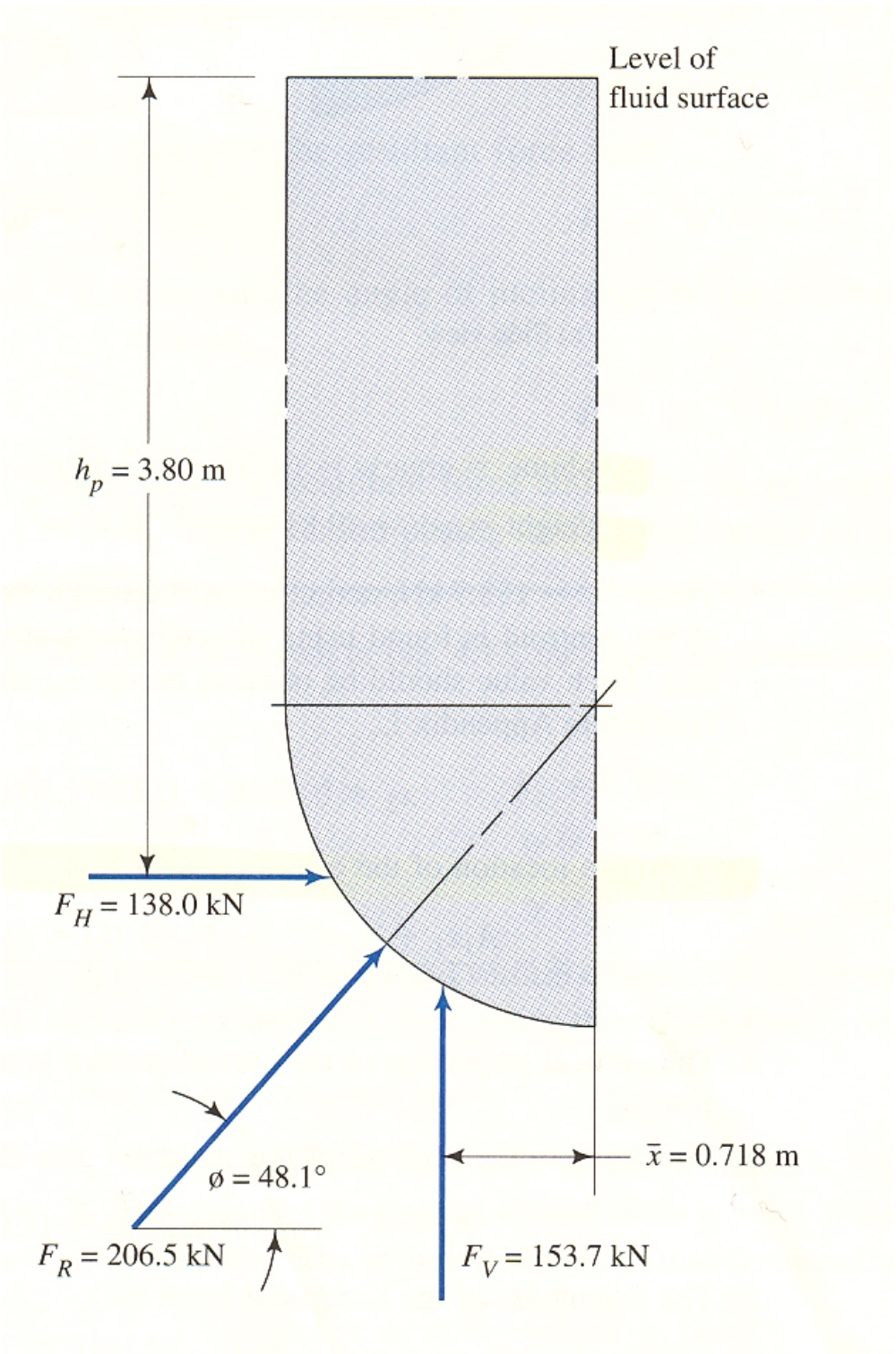
$= (153.7^2+138^2)^{1/2}$

= **206.5 kN**

Angle of inclination of the resultant force = $\tan^{-1}$ (Fv/Fh)

$= \tan^{-1}$ (153.7/138.0)

= **48.1°**


Level of
fluid surface
h_p = 3.80 m
F_H = 138.0 kN
ø = 48.1°
F_R = 206.5 kN
$\bar{x}$ = 0.718 m
F_V = 153.7 kN

Assignment # 4:

- 4.47M