CHAPTER ELEVEN

11. THREE HINGED ARCHES

These are Curved Structures which are in use since ancient times. These were mostly used in buildings and the abatements used to be very thick. As our analysis capacity increased due to faster computers, it is now possible to understand behaviour of arches for various support, load and material conditions. These days arch bridges either in Reinforced concrete or the pre-stressed concrete are becoming a common sight due to asthetics of curved surfaces.

Arches when loaded by gravity loads, exhibit appreciable compressive stresses. At supports, horizontal reaction (thrust) is also developed which reduces the bending moment in the arch.

Aches can be built in stone, masonry, reinforced concrete and steel. They can have a variety of end conditions like three hinged arches, two hinged arches and find arches. Considering the geometry these can be segmental, parabolic and circular. An arch under gravity loads generally exhibits three structural actions at any cross-section within span including shear force, bending moment and axial compressive force. The slope of centerline of arch keeps on varying along span so above mentioned three structural actions also vary along span.

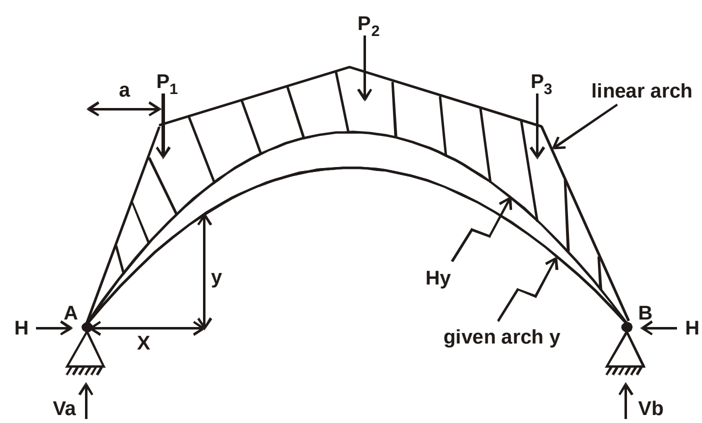
11.1. Eddy's theorem:

The bending moment at any point on the arch is the difference between simple span bending moment and product Hy".

Where H is the horizontal thrust at supports (springings), y is the rise of arch at a distance X from the origin.

Shape of simple span bending moment diagram due to applied loads is also called linear arch. Hy may also be termed as equation of centerline of actual arch multiplied by a constant (H).

Consider the following arch carrying the loads P_1, P_2 and P_3. The shaded area is the BMD.



Bending moment at X is

$$M_X = V_aX - Hy - P_1(X - a)$$

$$M_X = \mu X - Hy. \quad \text{(Eddy's theorem)}$$

Where $\mu_z = V_a \times x - P_1(X - a)$ = Simple span bending moment considering the arch to be a simple beam.

The inclined axial force (normal thrust) also contributes towards vertical shear force in addition to applied loads and reactions.

11.2. Three-hinged arch:

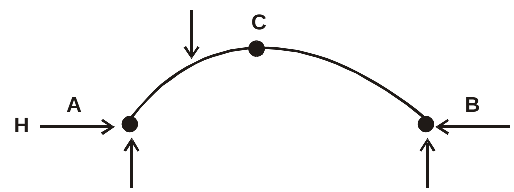
If an arch contains three hinges such that two hinges are at the supports and the third one anywhere within span, it is called a three hinged arch. This type of arch is statically determinate wherein reactions, horizontal thrust and all internal structural actions can be easily determined by using the laws of equilibrium and statics. If the third hinge is provided at the highest point, it is called crown of the arch.

Consider a three hinged arch with third hinge at the crown, then

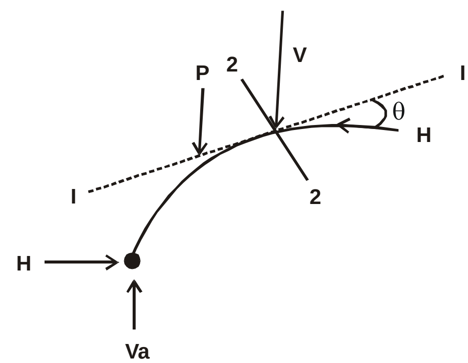
$M_X = \mu X - Hy$ (1) becomes at center

$M_c = \mu c - Hyc = 0$

SO $H = \frac{\mu_c}{Y_c}$ (2)



Cutting the arch as shown, and projecting forces along axis 1–1 and 2–2 and putting $V = V_a - P_1$ we have.



$P = H\cos\theta + V\sin\theta$ (3) along 1–1

$Q = H\sin\theta - V\cos\theta$ (4) along 2 – 2

11.3. Parabolic Arch

If a three-hinged parabolic arch carries udl over its span, the arch will carry pure compression and no SF or BM. This is because the shape of linear arch (BMD due to loads) will be the same as shape of actual arch.

For a parabolic arch having origin at either of springings, the equation of centre line of arch at a distance X from origin where rise is y will be.

$y = C.X(L - X)$ (5) constant C will be evaluated from boundary conditions.

at $X = \frac{L}{2}$, $y = y_c$. we get

$Y_c = C.\frac{L}{2}.\frac{L}{2}$ or $C = \frac{4y_c}{L^2}$

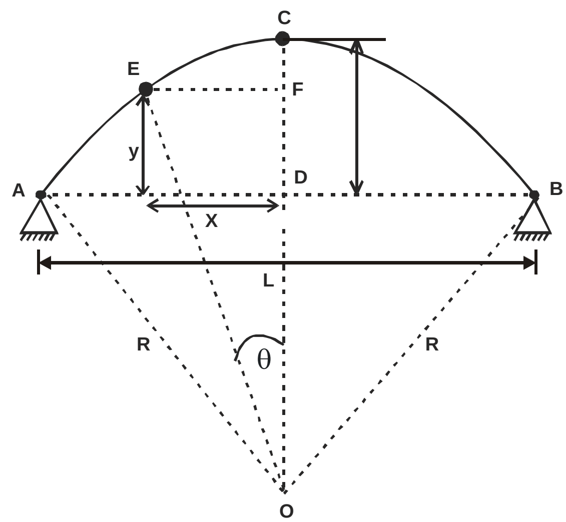
So $y = \frac{4y_c}{L^2} . X(L - X)$ (6)

The slope θ can be calculated from

$$\frac{dy}{dX} = \tan\theta = \frac{4\,y_c}{L^2}(L - 2X) \qquad (7)$$

11.4. Circular Arch:

If arch is a part of Circle, it is convenient to have origin at the centre.



Consider triangle OEF

$$OE^2 = EF^2 + OF^2$$

Or $$R^2 = X^2 + (R - y_c + y)^2 \qquad (8)$$

and we also have from triangle ADO

$$\frac{L^2}{4} + (R - y_c)^2 = R^2$$

$$y_c(2R - y_c) = \frac{L^2}{4} \qquad (9)$$

As span and central rise are usually known, Radius of arch R can be calculated from (9)

Equation (8) can bge written as $y = \sqrt{R^2 - X^2} - (R - y_c)$

Now once the basic equations for parabolic and circular arches have been established, let us solve some numericals.

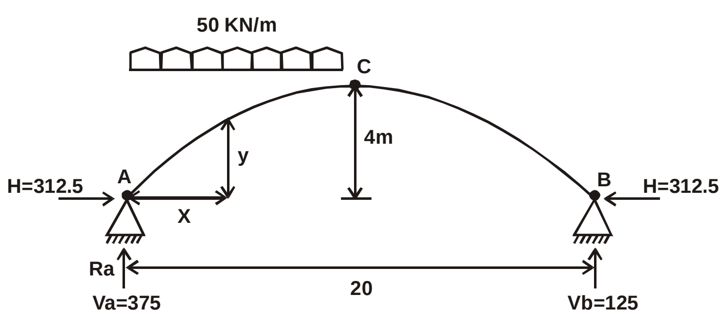
EXAMPLE NO. 1

Analyze a three-hinged arch of span 20m and a central rise of 4m. It is loaded by udl of 50 KN/m over its left half. Calculate maximum positive and negative moments if

(i) The arch is parabolic

(ii) The arch is circular

SOLUTION: 1. Arch is Parabolic



$\sum Ma = 0$

$Vb \times 20 = 50 \times 10 \times 5$

$V_b = \frac{2500}{20} = 125$ KN

$V_a + V_b = 50 \times 10 = 500$ KN

So $V_a = 500 - V_b = 500 - 125$

$= 375$ KN

$H = \frac{\mu_c}{y_c} = \frac{125 \times 10}{4} = 312.5 \text{ KN}$

$H = 312.5 \text{ KN}$

$R_a = \sqrt{Va^2 + H^2}$

$= \sqrt{375^2 + 312.5^2} = \sqrt{140625 + 9765.25}$

$= \sqrt{238281.25} = 488.14 \text{ KN}$

$\text{Tan}\theta a = \frac{Va}{H} = \frac{375}{312.5} = 1.2$

$\theta_a = 50.19^o$

and $R_b = \sqrt{Vb^2 + H^2} = \sqrt{125^2 + 312.5^2}$

$Rb = \sqrt{15625 + 97656.25}$

$Rb = \sqrt{113281.25} = 336.57 \text{ KN}$

$\text{Tan}\theta b = \frac{Vb}{H} = \frac{125}{312.5} = 0.4$

$\theta_b = 21.80^0$

Maximum positive Moment

It is expected in portion AC. Write generalize M_X expression.

$M_X = 375X - \frac{50X^2}{2} - 312.5y$

Now $y = \frac{4y_c}{L^2}(L - X) = \frac{4 \times 4}{20^2} X(20 - X) = 0.04\,(20X - X^2)$

$y = 0.8 - 0.04X^2$

So

$M_X = 375X - 25X^2 - 312.5\,[0.8X - 0.04X^2]$

$= 375X - 25X^2 - 250X + 12.5X^2$ Simplifying

$M_X = 125X - 12.5X^2$

$\frac{dM_X}{dX} = V_X = 0 = 125 - 25X$

$X = 5$m from A. Putting Value of X in M_X expression above.

So

$M_{max} = 125 \times 5 - 12.5 \times 5^2$

$= 625 - 312.5$

$M_{max} = 312.5$ KN-m

Maximum negative moment:

It would occur in portion BC at a distance x from B.

$M_X = 125X - 312.5y$, Putting equation of y.

$= 125X - 312.5\,(0.8X - 0.04X^2)$

$M_X = 125X - 250X + 12.5X^2$

$M_X = -125X + 12.5X^2$

$\frac{dMx}{dX} = V_X = 0 = -125 + 25X$

$X = 5_m$ from B.

So putting value of X in M_X expression above.

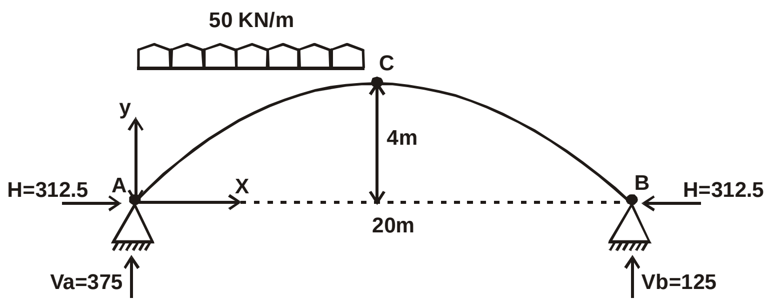
$M_{max} = -125 \times 5 + 12.5(5)^2$

$= -625 + 312.5$

$M_{max} = -312.5$ KN–m

SOLUTION: Considering Circular Arch

EXAMPLE NO.2: Now or Solve the following loaded three hinged Circular Arch



Step 1. Reactions:

As before reactions are same.

Step 2. Equation of Circular Arch

The general equation is $(X - h)^2 + (y - k)^2 = r^2$

h and k are co-ordinates at the centre and r is radius of Circle. There are three unknown in above equation, Viz, h, k and r and these can be determined from the following boundary conditions. Origin is at point A.

Boundary conditions

1. At $X = 0$, $y = 0$ It gives $(-h)^2 + (-k)^2 = r^2$

   $h^2 + k^2 = r^2$ (1)

2. At $X = 20$, $y = 0$ It gives $(20 - h)^2 + (-k^2) = r^2$

   $400 + h^2 - 40h + k^2 = r^2$ (2)

3. At $X = 10$, $Y = 4$ It gives $(10 - h)^2 + (4 - k)^2 = r^2$

   $100 + h^2 - 20h + 16 + k^2 - 8k = r^2$

   $116 + h^2 - 20h + k^2 - 8k = r^2$ (3)

Subtract (1) from (2) we get

$400 - 40h = 0$

Or $h = 10$

Put value of h in (1) and 3

$100 + k^2 = r^2$ (1)

$116 + 100 - 200 + k^2 - 8k = r^2$ (3)

or $16 + k^2 - 8k = r^2$ (3)

$16 + k^2 - 8k = 100 + k^2$ (by putting Value of r^2 from 1)

$8k = 16 - 100 = -84$

$$k = \frac{-84}{8} = -10.5$$

Putting k = – 10.5 in (3)we get

$r^2 = 16 + (-10.5)^2 + 8 \times 10.5$

$= 16 + 110.25 + 84 = 210.25$

So r = 14.5 meters.

Putting Values of h, k and r in general equation, we get

$(X - 10)^2 + (y + 10.5)^2 = 14.5^2$ Simplify it, we get.

$$y = -10.5 + \sqrt{14.5^2 - (X - 10)^2}$$

$(y + 10.5)^2 = 14.5^2 - (X - 10)^2$

$$= -10.5 + \sqrt{210.25 - X^2 - 100 + 20X}$$

$$y = -10.5 + \sqrt{110.25 - X^2 + 20X} \qquad (4)$$

We know, $yc\,(2r - yc) = \dfrac{L^2}{4}$ (5)

and

$$y = \sqrt{r^2 - \left(\frac{L}{2} - X\right)^2} - (r - yc) \qquad (6)$$

These equations are same as were used in derivation earlier.

Alternatively to avoid evaluation of constants each time, equations (5) and (6) can be used. Equation (6) is the equation of Centre-line of Circular arch.

Step 3: Calculation of Maximum moment.

Maximum positive moment occurs in span AC. Write M_X expression

$$M_X = 375X - \frac{50X^2}{2} - 312.5\,y$$ put y from (4) above.

$$= 375X - 25X^2 - 312.5\left[-10.5 + \sqrt{110.25 - X^2 + 20X}\right]$$

$$M_X = 375X - 25X^2 + 3281.25 - 312.5\sqrt{110.25 - X^2 + 20X}$$

Now maximum moment occurs where shear force is zero. So

$$\frac{dM_X}{d_X} = V_X = 375 - 50X - \frac{312.5\,(-2X + 20)}{2\sqrt{110.25 - X^2 + 20X}} = 0$$

$$375 - 50X = \frac{312.5\,(-X + 10)}{\sqrt{110.25 - X^2 + 20X}} \qquad \text{divide by 50}$$

$$7.5 - X = \frac{6.25\,(10 - X)}{\sqrt{110.25 - X^2 + 20X}} \qquad \text{multiply by } -1, \qquad \text{We get}$$

$$X - 7.5 = \frac{6.25\,(X - 10)}{\sqrt{110.25 - X^2 + 20X}}$$

$$(X - 7.5)\sqrt{110.25 - X^2 + 20X} = 6.25\,(X - 10) \qquad \text{square both sides}$$

$$(X - 7.5)^2\,(110.25 - X^2 + 20X) = 6.25^2\,(X - 10)^2 \qquad \text{Simplify}$$

$$(X^2 - 15X + 56.25)\,(110.25 - X^2 + 20X) = 39.0625\,(X^2 - 20X + 100)$$

or $$110.25X^2 - X^4 + 20X^3 - 1653.75X + 15X^3 - 300X^2 = 39.0625X^2 - 781.25X + 3906.25$$

$$+ 6201.56 - 56.25X^2 + 1125X$$

Simplifying

$$-X^4 + 35X^3 - 285.0625X^2 + 252.5X + 2295.3125 = 0$$

or $$X^4 - 35X^3 + 285.0625X^2 - 252.5X - 2295.3125 = 0$$

Now it is considered appropriate to solve this equation by Modified Newton – Raphson iteration solutions which in general is

$$X_{n+1} = X_n + \frac{f\,(x_n)}{f^{/}\,(X_n)} \qquad \text{(A)}$$

So $f\,(X) = X^4 - 35X^3 + 285.0625X^2 - 252.5X - 2295.3125$

And differentiate, $f^{/}\,(X) = 4X^3 - 105X^2 + 570.125X - 252.5$

In general, it is recommended that first root X_n should be always taken at 1 because it converges very fast. However, knowing that B. M will be maximum near the middle of portion AC, we take $X_n = 2$ (to reduce number of iterations possibly) and solve in the following tabular form. Evaluate $f(X)$ and $f^{/}(X_n)$ expressions.

Iteration Number	X_n	$f(X_n)$	$f^{/}\,(X_n)$	X_{n+1} from A above
1	2	–1924.06	499.75	5.85
2	5.85	147.251	290.1629	5.3425
3	5.3425	– 30.3142	406.3845	5.417
4	5.417	– 0.58794	390.546	5.418

So we get X_n and $X_n + 1$ as same after 4th iteration.

So X = 5.418 m put this in MX expressions

$M_{max} = 375\,(5.418) - 25\,(5.418)^2 + 3281.25 - 312.5\sqrt{110.25 - 5.418^2 + 20 \times 5.418}$

$= 280.066$ KN-m

Maximum negative moment in the arch

Let us assume that it occurs in portion BC at a distance X from A $(10 < X < 20)$

$M_X = 125\,(20 - X) - 312.5\,(-10.5 + \sqrt{110.25 - X^2 + 20X}\,)$ Simplify

$= 2500 - 125X + 3281.25 - 312.5\sqrt{110.25 - X^2 + 20X}$

or $M_X = 5781.25 - 125X - 312.5\sqrt{110.25 - X^2 + 20X}$

Maximum moment occurs where SF is zero, So differentiate M_X expression w.r.t. X.

$$\frac{dM_X}{dX} = 0 = -125 - \frac{312.5\,(-2X + 20)}{2\sqrt{110.25 - X^2 + 20X}}$$

or $$0 = -125\ \frac{+\ 312.5(X - 10)}{\sqrt{110.25 - X^2 + 20X}}$$

$125\sqrt{110.25 - X^2 + 20X} = 312.5\,(X - 10)$ squaring both sides. We have,

$15625\,(110.25 - X^2 + 20X) = 97656.25\,(X^2 - 20X + 100)$ Simplify

$110.25 - X^2 + 20X = 6.25\,(X^2 - 20X + 100)$

$0 = 7.25X^2 - 145X + 514.75$ dividing by 7.25

$X^2 - 20X + 71 = 0$ Solve this quadretic equation.

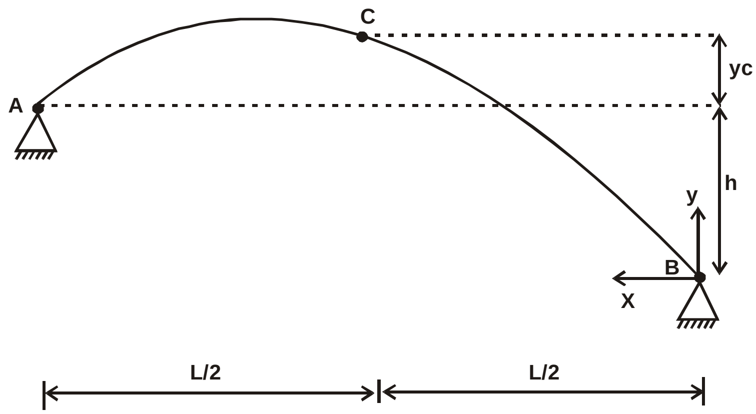
$$X = \frac{20 \pm \sqrt{400 - 284}}{2}$$

$X = \dfrac{20 \pm 10.77}{2} = 15.385$m from A Put this value of X in M_X expression above.

So $M_{max} = 5781.25 - 125 \times 15.385 - 312.5\sqrt{110.25 - (15.385)^2 + 20\,(15.385)}$

$= 5781.25 - 1923.125 - 312.5\sqrt{181.257} = -349.115$KN.m

11.5. Derivation for center-line of a parabolic arch with supports at different levels.



The general form of a parabola is

$$y = aX^2 + bX + c$$

Evaluate constants a, b and c by putting boundary conditions in above equation

At $X = o$; $Y = o$, (Point B) So $C = o$ (1)

At $X = L$; $Y = h$, (Point A) So $h = aL^2 + bL$ (2)

At $X = \frac{L}{2}$; $Y = y_c + h$, (Point C) So $y_c + h = \frac{aL^2}{4} + \frac{bL}{2}$ (3) multiply by 4

$$h = aL^2 + bL \quad (2)$$

Equation (3) can also be written as

$4(yc + h) = aL^2 + 2bL$ (3) Subtract (3) from (2), we have

$h - 4(y_c + h) = -bL$ or $b = \frac{4}{L}(y_c + h) - \frac{h}{L}$

Put this value of b in (2) and solve for a

$h = aL^2 + 4(y_c + h) - h$ or $a = \frac{2h - 4(yc + h)}{L^2}$)

$$a = \frac{-2h - 4y_c}{L^2}$$

Now all constant have been evaluated in general terms. Put Values of a, b and c in general equation; we have

$y = \dfrac{-2X^2(h + 2y_c)}{L^2} + \dfrac{X(4y_c + 3h)}{L}$. This is the generalized equation for a parabolic arch with supports at different levels. Test this derived equation and see whether boundary conditions are satisfied.

– At $X = o$; $y = o$, put this in above equation. It is satisfied
– At $X = L$, $y = h$, put this in above equation. It is satisfied
– At $X = \dfrac{L}{2}$, $y = h + y_c$, put this in above equation. It is also satisfied.

If supports are at the same level, $h = o$ Put this in above equation, we get

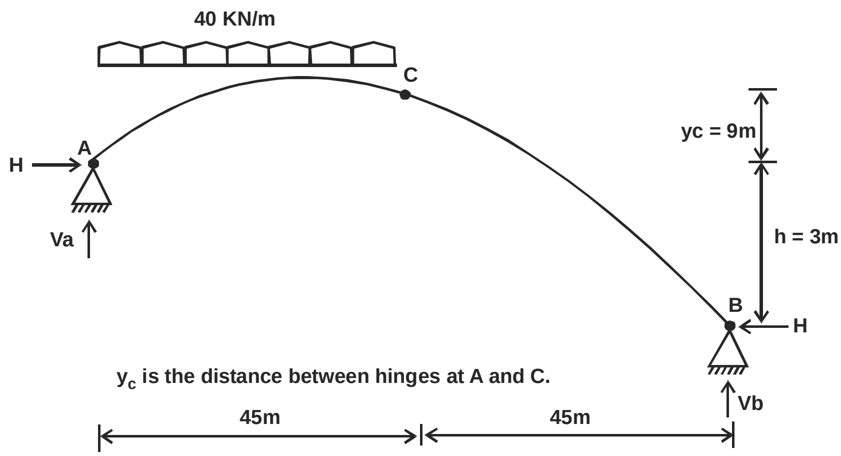
$y = \dfrac{-4y_cX^2}{L^2} + \dfrac{4y_cX}{L}$ or $y = \dfrac{4y_cX}{L^2}(L - X)$, after simplification.

and $\dfrac{dy}{dX} = \dfrac{4y_c}{L^2}(L - 2X)$

These two equations have already been used. Now we solve some Example.

EXAMPLE NO.3:-

Solve the following 3 hinged parabolic loaded arch with supports at different levels as shown.



$\sum Fy = 0$

$V_A + V_b = 40(45) = 1800$ KN (1)

$\sum Mc = O$, $VA(45) - 9H - 40(45)\left(\dfrac{45}{2}\right) = 0$ (2) Moments at C of forces on its left.

$45V_A - 9H - 40500 = 0$

$45V_b - 12H = 0$ (3) Moment at C of forces on its right

Divide Equation (2) by 9

$5VA - H - 4500 = 0$ (2)

Multiply this Equation by 12 and subtract equation (3) from it.

$60\,V_A - 12H - 54000 = 0$ (2)

$45V_B - 12H = 0$ (3)

$60\,V_A - 54000 - 45V_B = 0$ (4)

Multiply Equation (1) by 45 and add in equation (4)

$45V_A + 45V_B = 81000$ (1)

$60\,V_A - 45V_B = 54000$

Adding we get.

$105\,V_A = 135000$

or $V_A = \dfrac{135000}{105}$

$V_A = 1285.7$ KN , put this value in equation (1)

so $V_B = 514.3$ KN

We know, $45V_B - 12H = 0$ (3) , from this

$H = \dfrac{45 \times V_B}{12}$

$= 1928.63$ KN (after putting value of V_b)

$H = 1928.63$ KN

New calculate $(M_{ac})_{max}$ and $(M_{bc})_{max}$

Keeping " B" as origin , at a distance X from B in portion AC moment expression is

$(M_{ac}) = Va\,(90 - X) - H(y) - \dfrac{40}{2}(90 - X)^2$

$y = \dfrac{X\,(4yc + 3h)}{L} - \dfrac{2X^2\,(3 + 18)}{90^2}$. This equation was derived in previous article.

If h $= 3$m , $y = \dfrac{X}{2} - \dfrac{7X^2}{1350}$ (A) If h = 0 , $y = 0.4X - \dfrac{X^2}{225}$ (B)

$(M_{ac}) = 1285.7\,(90 - X) - 1928.63\left[0.4X - \dfrac{X^2}{225}\right] - 20\,(90 - X)^2$ – (i)

after putting values of h and y_c in above equation for y.

$= 115713 - 1285.7X - 771.45X + 8.57X^2 - 20\,(8100 + X^2 - 180X)$

$\dfrac{dMac}{dX} = 0 = -1285.7 - 771.45 + 17.14X - 40X + 3600$ Simplify

$0 = + 1542.85 - 22.86X$

$X = 1542.85/22.86 = 67.5$m (This value should be more than 45)

Putting this Value in Equation (i)

$(M_{ac})_{max}$ = $1285.7\ (22.5) - 1928.63\ (27 - 20.25) - 20\ (22.5)^2$

= 5785 KN-m

$(M_{bc})_{max} = 514.3X - 1928.63 \times y$. Moment at a distance X from B.

$= 514.3X - 1928.63 \times \left[\frac{X}{2} - \frac{7X^2}{1350}\right]$ (After putting equation for y) and values of yc, h and L and using equation A.

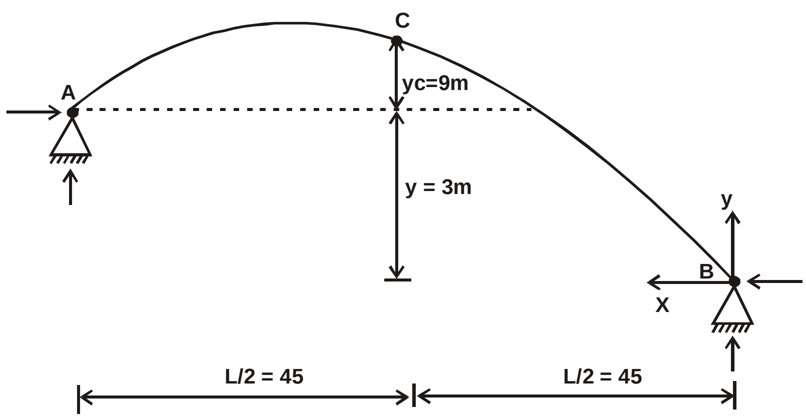
$= 514.3X - 964.315X + 10X^2$

$\frac{dM_{bc}}{dX} = \quad 0 = 514.3 - 964.315 + 20X$

$= 20X = 450$

$X = 22.5m$, $M_{bc} = -5062.68$ KN–m (after putting value of X above)

11.6. Development of Generalized equation of three hinged circular arch with support at different levels.



General Equation of Circle is

$(X - h)^2 + (y - k)^2 = R^2$

at X = 0, y = 0 $\sim h^2 + k^2 = R^2$ – (1)

at X = 45 , $y = y_c + \delta = 12$

Putting $(45 - h)^2 + (12 - k)^2 = R^2$ Simplifying it.

$2025 - 90h + h^2 + 144 - 24k + k^2 = R^2$

$2025 + 144 - 90h - 24k + h^2 + k^2 = R^2$ (2) Simplifying

$2169 - 90h - 24k + h^2 + k^2 = R^2$ (2)

at X = 90 , y = 3 [point A]

$(90 - h)^2 + (3 - k)^2 = R^2$ Simplifying

Put these values, $8100 - 180h + h^2 + 9 + k^2 - 6k = R^2$

$8109 - 180h - 6k + h^2 + k^2 = R^3$ (3)

Equating (1) with (2) and multiply resulting equation by 2 and then equation (1) and (3)

$2\,[2169 - 90h - 24k = 0]$ ~ (4) or $4338 - 180h - 48k = 0$ (4)

$8109 - 180h - 6k = 0$ ~ (5)

Subtract (4) from (5), we have

$$3771 + 42k = 0$$

$$k = \frac{-3771}{42}$$

$$k = -89.79$$

Put in Eq (4)

$$2169 - 90h - 24(-89.79) = 0$$

$$h = \frac{4323.86}{90}$$

$$h = 48.04$$

Now from (1)

$$(48.04)^2 + (-89.79)^2 = R^2$$

$$R = 101.83\text{m}$$

Now write equation of center-line of arch.

$$y = \sqrt{R^2 - (X - h)^2} + k$$

$$= \sqrt{(101.83)^2 - (X - 48.04)^2} + (-89.79)$$

$$y = \sqrt{(10369.35 - X^2 - 2307.84 + 96.08X)} - 89.79 \quad \text{(A)}$$

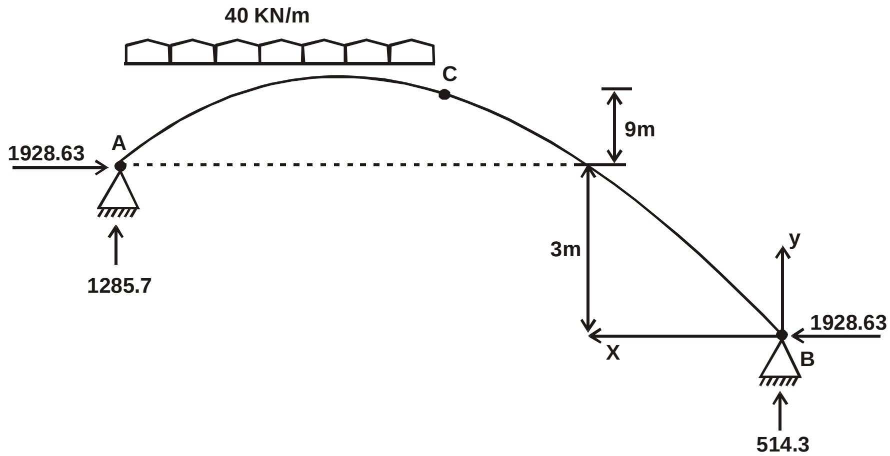
Point B: At X = 0 , y = 0 (see diagram now)

Point C: At X = 45 , y = 12

Point A: At X = 90 , y = 3

So Eq ,(A) has been correctly derived.

EXAMPLE NO. 4: Calculate maximum moments in portion AC & BC for the following 3-hinged loaded Circular each.



SOLUTION: Reactions will be same as Previous.

1. Calculation of (Mac)max.

Write moment expression for use previously developed equation. Consider forces on left of section.

$$M_X = 1285.7\,(90 - X) - \frac{40}{2}(90 - X)^2 - 1928.63\,(\sqrt{10369.35 - X^2 - 2307.84 + 96.08X} - 89.79)$$

$$M_X = 115713 - 1285.7X - 20(8100 - 180X + X^2) + 173171.7 - 1928.63\sqrt{(10369.35 - X^2 - 2307.84 + 96.08X)}$$

$M_X = 126884.69 - 20X^2 + 2314.3X - 1928.63\,(8061.51 - X^2 + 96.08X)^{1/2}$ - (B) differentiate w.r.t.

$$\frac{dM_X}{dX} = 0 = -40X + 2314.3 - \frac{964.315\,(-2X + 96.08)}{(8061.51 - X^2 + 96.08X)^{1/2}}$$

$$(40X - 2314.3)\,(8061.51 - X^2 + 96.08X)^{1/2} = 964.315\,(2X - 96.08)$$

$$(40X - 2314.3)^2\,(806151 - X^2 + 96.08X) = [1928.63\,(X - 48.04)]^2$$

Squaring and simplifying, we get.

$(1600X^2 + (2314.3)^2 - 185144X)\,(8061.51 - X^2 + 96.08X) = 1928.632^2\,(X^2 + 2307.84 - 96.08X)$ Simplifying

$1298416X^2 - 1600X^4 + 153728X^3 + 4.32 \times 10^{10} - 5355984.5X^2 + 514602989.8X$

$-1.49254021 \times 10^9 X + 185144X^3 - 17.78863 \times 10^6 X^2 = 3719613.68X^2 + 8.5843 \times 10^9 - 357380482.1X$ Simplifying

$-13965812.2X^2 - 1600X^4 + 338872X^3 - 620556738X + 3.46157 \times 10^{10} = 0$

$8728.63X^2 + X^4 - 211.8X^3 + 387848X - 2163412.5 = 0$

$f(X) = X^4 - 211.8X^3 + 8728.63X^2 + 387848X - 21634812.5$ differentiate it.

$f'(X) = 4X^3 - 635.4X^2 + 17457.26X + 387848$

To cut-short, Let X = 55 (Because it is portion AC and X has to be more than 45)

So $f(X) = (55)^4 - 211.8 \times (55)^3 + 8728.63 \times (55)^2 + 387848\,(55) - 21634812.5$

$= 1333.25$

$f'(X) = 4(55)^3 - 635.4(55)^2 + 17457.26(55) + 387848$

$= 91412.3$

$$X_n - \frac{fx}{f'(x)} = X_{n+1}$$

$$= 55 - \frac{13333.25}{91412.3}$$

$= 55 - 0.146$

$= 54.85$

Now $X = 54.85$ (use this update value now)

$f(X) = -560.16$

$f'(X) = 93833.35$

$$X_{n+1} = Xn - \frac{f(n)}{f'(n)}$$

$$= 54.85 - \frac{(-560.16)}{93833.35}$$

X_{n+1} or $X = 54.855969$. The value of X has converged now.

Putting this value of X in Equation B to find $(Mac)_{max}$

$$(Mac)_{max} = 126884.69 - 20\,(54.855969)^2 + 2314.3\,(54.855969)$$
$$- 1928.63\,(8061.51 - (54.855969)^2 + 96.08 \times 54.855969)^{1/2}$$

$(Mac)_{max} = 193552.7$ KN–m

$(Mbc)_{max}$. Working on similar lines $(M_{bc})_{max}$ can be calculated now.

$M_X = 514.3X - 1928.63\,y$ putting equation of center line of arch y.

$$= 514.3X - 1928.63\,[\sqrt{10369.35 - X^2 - 2307.84 + 96.08X} - 89.79] \qquad (C)$$

$$\frac{dM_X}{dX} = 0 = 514.3 - \frac{1928.63}{2} \times \frac{(-2X + 96.08)}{(10369.35 - X^2 - 2307.84 + 96.08X)^{1/2}} = 0$$

or $$514.3 + \frac{1928.63\,(X - 48.04)}{(8061.51 - X^2 + 96.08X)^{1/2}} = 0 ,$$

$$-514.3\,(8061.51 - X2 + 96.08X)^{1/2} = 1928.63\,(X - 48.04)$$

Squaring both sides of equation.

$$264504.5\,[\,(8061.51 - X^2 + 96.08X)^{1/2}]^2 = [1928.63\,(X - 48.04)]^2$$

we get.

$264504.5\ (8061.51 - X^2 + 96.08X) = (1928.63)^2 (X^2 + 2307.84 - 96.08X)$ Simplifying

$2132305672 - 264504.5X^2 + 25413592.36X = 3719613.68X^2 + 8584273228 - 357380482.1X$

$- 6451967556 - 3984118.18X^2 + 382794074.1$ on further simplification, we get

$- X^2 + 96.08X - 1619.422 = 0$ (after dividing by 3984118.18)

or $X^2 - 96.08X + 1619.422 = 0$ solving this quadretic equation where.

$a = 1,\ b = -96.08,\ C = 1619.422$

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{96.08 \pm \sqrt{(96.08)^2 - 4(1)\ (1619.422)}}{2}$$

$$= \frac{96.08 \pm 52.47}{2}$$

$$X = \frac{96.08 + 52.47}{2} \quad \text{Or} \quad \frac{96.08 - 52.47}{2}$$

$$= 74.24 \quad \text{or} \quad 1.80$$

Therefore, (X = 74.24 is not applicable so not accepted as a root.

$$X = 21.80\text{m}$$

Put this value of X in equation (C), we have $(Mbc)_{max}$.

Putting in (c)

$$(Mbc)_{max} = 514.3X - 1928.63\sqrt{10369.35 - X^2 - 2307.84 + 96.08X} + 173171.69$$

$$= 11211.74 - 189698.0911 + 173171.69$$

$$= - 5314.67 \text{ KN-m}$$

We have solved some representative problems. Using the guidance given in this chapter a student should be able to solve any problem on three hinged parabolic or circular arches, whether supports are at the same level or not.