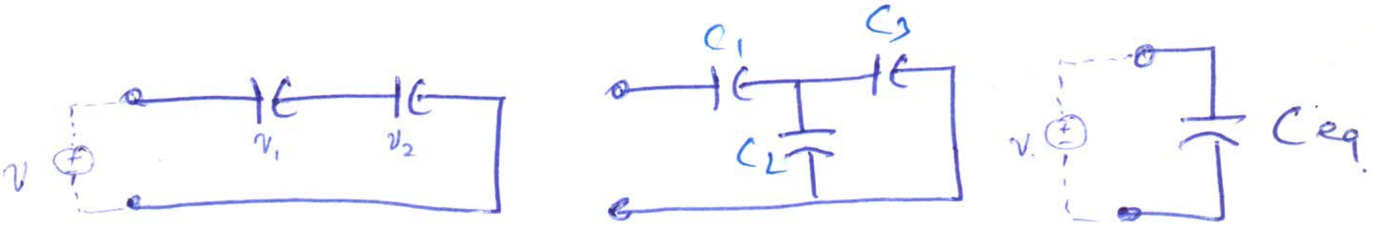
Problems.

3.1 what must be the relationship between C_{eq} and C_1 and C_2 in (a) of the figure of the network if (a) and (c) are equivalent ?.
Repeat for the network shown in (b).



(a) (b) (c)

$\boxed{a}$ fig: (a) equivalent to fig: (c)

According to K.V.Law.

$$\Sigma \text{ V. rise} = \Sigma \text{ Voltage-drops.}$$

$$V = V_1 + V_2$$

from fig: (c)

$$\frac{q}{C_{eq}} = \frac{q}{C_1} + \frac{q}{C_2} \qquad \left[V_1 = \frac{q}{C_1} , V_2 = \frac{q}{C_2} \right]$$

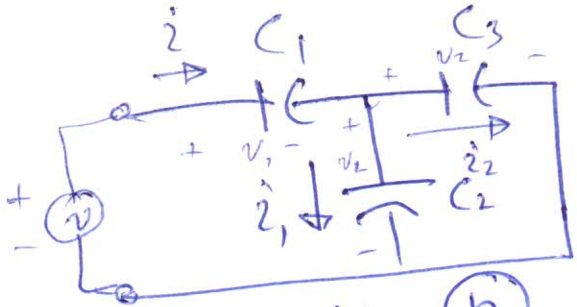
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}.$$

or

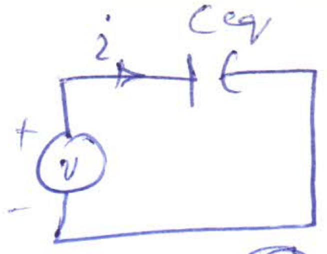
$$\frac{1}{C_{eq}} = \frac{C_1 + C_2}{C_1 C_2}$$

$$\boxed{C_{eq} = \frac{C_1 C_2}{C_1 + C_2}}$$

b



(b)



(C)

$$i = i_1 + i_2$$

$$i_1 = C_1 \frac{dv_1}{dt}, \quad i_1 = C_2 \frac{dv_2}{dt}, \quad i_2 = C_3 \frac{dv_3}{dt}.$$

where v_1 = voltage across C_1, v_2 = across C_2 and v_3 = voltage across C_3.

$$C_1 \cdot \frac{dv_1}{dt} = C_2 \frac{dv_2}{dt} + C_3 \frac{dv_3}{dt}$$

$$v_2 = v_3 = v_2 \text{ (say)}$$

i.e. $$\frac{dv_2}{dt} = \frac{dv_3}{dt}.$$

$$C_1 \cdot \frac{dv_1}{dt} = (C_2 + C_3) \frac{dv_2}{dt}.$$

So

$$\boxed{\frac{dv_2}{dt} = \frac{C_1}{C_2 + C_3} \cdot \frac{dv_1}{dt}} \longrightarrow (1)$$

from fig. (b):

$$v = v_1 + v_2.$$

$$\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt}$$

But the value of $\frac{dv_2}{dt}$ from previous equation(1) is ⇒

$$\frac{dv_2}{dt} = \frac{C_1}{C_2+C_3} \cdot \frac{dv_1}{dt}.$$

$$\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{C_1}{C_2+C_3} \cdot \frac{dv_1}{dt}.$$

$$= \frac{dv_1}{dt}\left(1 + \frac{C_1}{C_2+C_3}\right) = \frac{dv_1}{dt}\left(\frac{C_2+C_3+C_1}{C_2+C_3}\right)$$

So

$$\boxed{\frac{dv_1}{dt} = \frac{C_2+C_3}{C_1+C_2+C_3}\frac{dv}{dt}} \longrightarrow (2)$$

from fig. (b) $\quad i = C_1\frac{dv_1}{dt},$

from fig. (c) $\quad i = C_{eq}.\frac{dv}{dt}.$

$$i = i.$$

$$C_1\frac{dv_1}{dt} = C_{eq}.\frac{dv}{dt}.$$

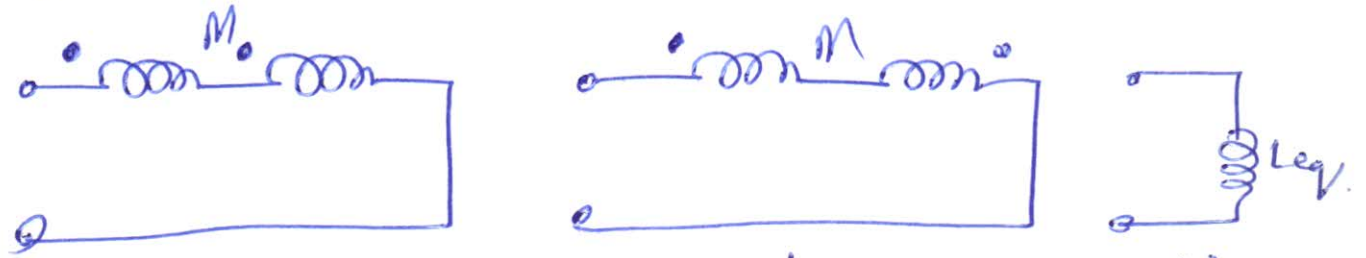
$$\boxed{\frac{dv_1}{dt} = \frac{C_{eq}}{C_1}.\frac{dv}{dt}} \longrightarrow (3)$$

Put (3) into equation (2)

$$\frac{C_{eq}}{C_1}\cdot\frac{d\cancel{V_1}}{dt} = \frac{C_2+C_3}{C_1+C_2+C_3}\,\frac{d\cancel{V}}{dt}$$

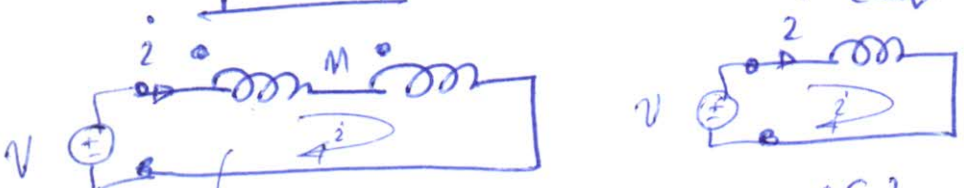
$$\boxed{C_{eq} = \frac{C_1\,(C_2+C_3)}{C_1+C_2+C_3}}$$

Prob: 3.2 What must be the relationship between L_{eq} and L_1, L_2 and M for networks of (a) and of (b) to be equivalent to that of (c)?



(a) (b) (c)

Part I



(a) (C)

Apply K.V.Law from fig. (a)

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + M_{12}\frac{di}{dt} + M_{21}\cdot\frac{di}{dt}$$

Now let $M_{12} = M_{21} = M$

$$\boxed{v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + 2M\frac{di}{dt}} \longrightarrow (1)$$

from fig. (c)

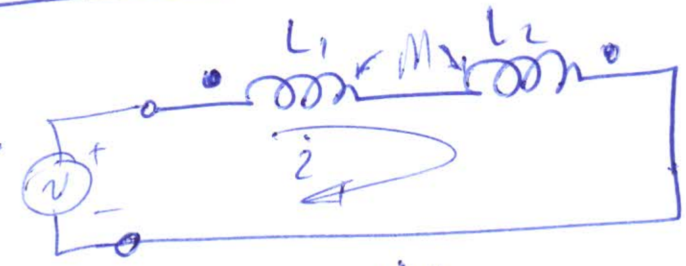
$$\boxed{v = L_{eq}\cdot\frac{di}{dt}} \longrightarrow (2)$$

Comparing ① & ②

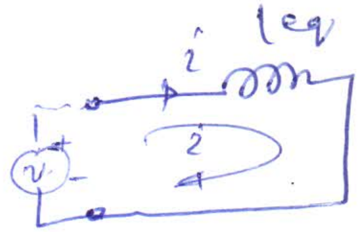
$$L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + 2M \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

$$\boxed{L_{eq} = L_1 + L_2 + 2M}$$

Part - II



(b)



(c)

from (b), Apply K.V.Law.

$$V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} - M_{12} \frac{di}{dt} - M_{21} \frac{di}{dt}$$

Let $M_{12} = M_{21} = M$.

$$\boxed{V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} - 2M \frac{di}{dt}} \rightarrow ③$$

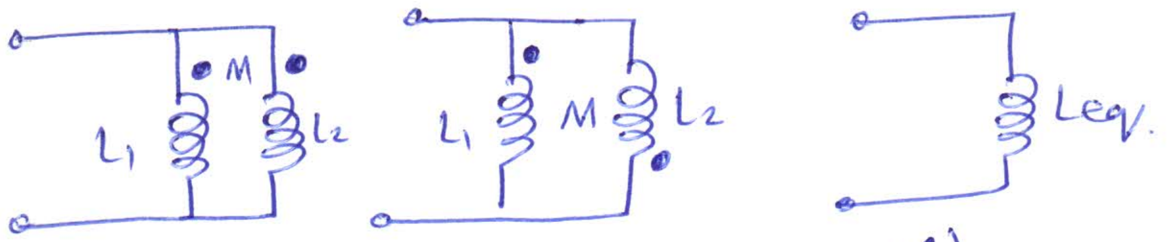
from ⓒ

$$\boxed{V = L_{eq} \frac{di}{dt}} \rightarrow ④$$

Equating ③ & ④

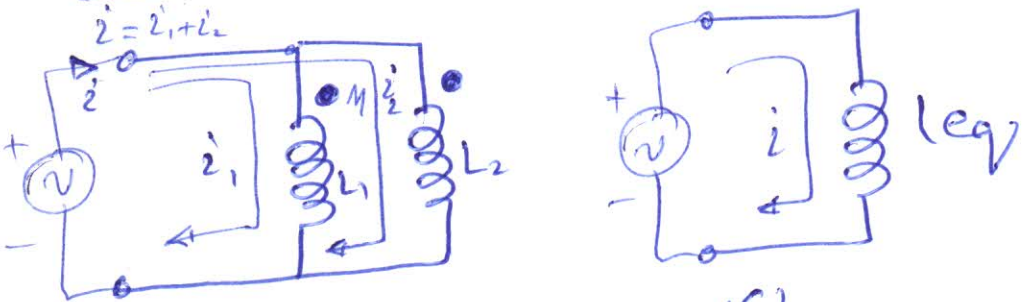
$$\boxed{L_{eq} = L_1 + L_2 - 2M}$$

Prob: 3.3 Repeat Prob: 3.2 for the three networks shown in the accompanying figure.



(a) (b) (c),

Part - I



(a) (c)

$$V = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad \text{--- (1)}$$

$$V = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad \text{--- (2)}$$

Equations (1) and (2)

$$L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\frac{di_2}{dt}(L_2 - M) = \frac{di_1}{dt}(L_1 - M)$$

i.e. $$\boxed{\frac{di_2}{dt} = \left(\frac{L_1 - M}{L_2 - M}\right) \frac{di_1}{dt}}$$

P.T.O

from fig: (c) $\quad v = L_{eq} \cdot \frac{di}{dt}$

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$v = L_{eq}\left[\frac{di_1}{dt} + \frac{di_2}{dt}\right] \longrightarrow (3)$$

from (2) $\quad v = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$

So, (3) becomes,

$$L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = L_{eq}\left[\frac{di_1}{dt} + \frac{di_2}{dt}\right]$$

Put $\frac{di_2}{dt} = \left(\frac{L_1 - M}{L_2 - M}\right)\frac{di_1}{dt}$

$$L_2 \left\{\frac{L_1 - M}{L_2 - M}\right\} \frac{di_1}{dt} + \frac{M\, di_1}{dt} = L_{eq}\left[\frac{di_1}{dt} + \left(\frac{L_1 - M}{L_2 - M}\right)\frac{di_1}{dt}\right]$$

or,
$$L_2\left(\frac{L_1 - M}{L_2 - M}\right) + M = L_{eq}\left[1 + \frac{L_1 - M}{L_2 - M}\right]$$

or
$$L_2\left(\frac{L_1 - M}{L_2 - M}\right) + M = L_{eq}\left[\frac{L_1 + L_2 - 2M}{L_2 - M}\right]$$

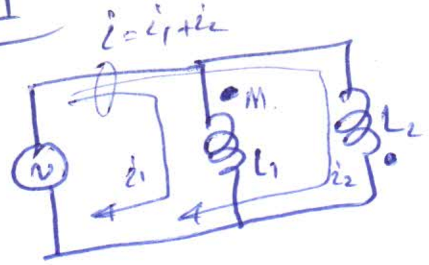
or
$$\frac{L_2(L_1 - M) + M(L_2 - M)}{L_2 - M} = L_{eq}\left(\frac{L_1 + L_2 - 2M}{L_2 - M}\right)$$

or
$$L_{eq} = \frac{L_1L_2 - L_2M + L_2M - M^2}{L_1 + L_2 - 2M}$$

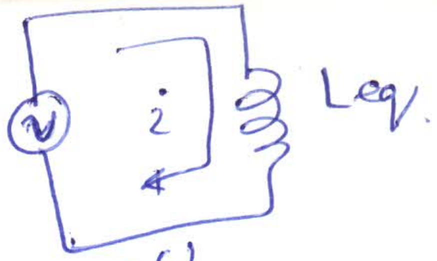
Finally

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

Part - II



(b)



(c)

Loop-1, $v = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$ — (1)

Loop-2 $v = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$ — (2)

Equating (1) and (2)

$$L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

or

$$\frac{di_2}{dt}(L_2 + M) = \frac{di_1}{dt}(L_1 + M)$$

or

$$\boxed{\frac{di_2}{dt} = \left(\frac{L_1 + M}{L_2 + M}\right)\frac{di_1}{dt}}$$

from fig: (c) $v = L_{eq} \cdot \frac{di}{dt}$

But $i = i_1 + i_2$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

so

$$v = L_{eq}\left[\frac{di_1}{dt} + \frac{di_2}{dt}\right] \rightarrow (3)$$

Also from (1)

$$v = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}.$$

Equating 3 & 2.

$$L_2 \frac{di_2}{dt} - M\frac{di_1}{dt} = L_{eq}\left[\frac{di_1}{dt} + \frac{di_2}{dt}\right] \rightarrow (4)$$

But we know that

$$\frac{di_2}{dt} = \left(\frac{L_1+M}{L_2+M}\right)\frac{di_1}{dt}$$

Put this in 4

$$L_2 \cdot \left(\frac{L_1+M}{L_2+M}\right)\frac{di_1}{dt} - M\frac{di_1}{dt} = L_{eq}\left[\frac{di_1}{dt} + \left(\frac{L_1+M}{L_2+M}\right)\frac{di_1}{dt}\right]$$

or ($\frac{di_1}{dt}$ is common on both side hence eliminate)

$$L_2\frac{(L_1+M)}{L_2+M} - M = L_{eq}\left[1 + \frac{L_1+M}{L_2+M}\right]$$

or

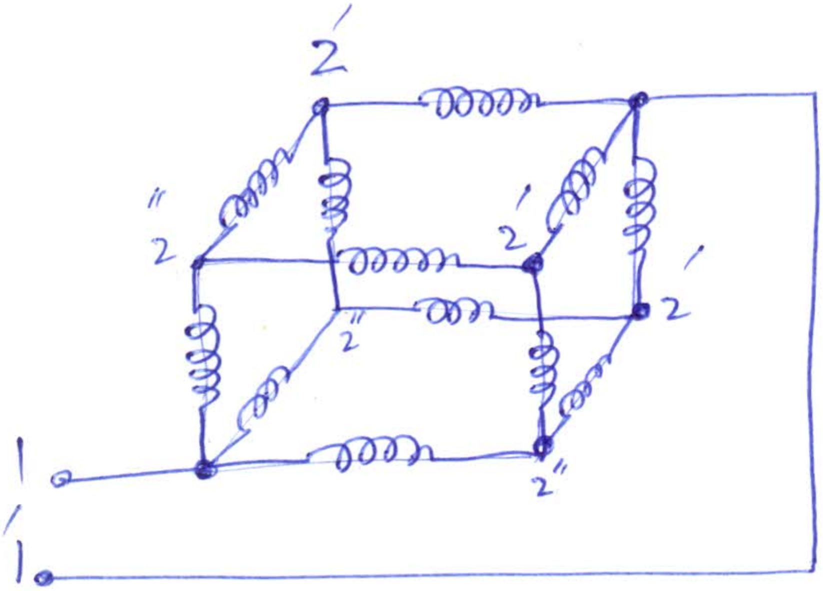
$$\frac{L_2(L_1+M) - M(L_2+M)}{L_2+M} = L_{eq}\left[\frac{L_2+M+L_1+M}{L_2+M}\right]$$

$$L_{eq} = \frac{L_1L_2 + L_2M - L_2M - M^2}{L_1+L_2+2M}$$

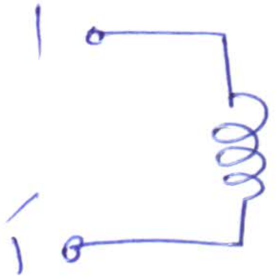
or

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1+L_2+2M}}$$

Prob: 3.4 The network of inductors shown in fig. is composed of a 1-H inductor on each edge of a Cube as shown. Show that, with respect to Vertices ⓐ and ⓑ, the network is equivalent to that in ⓑ of the figure when $L_{eq} = \frac{5}{6}H$.
Make use of symmetry in working in this problem, rather than writing Kirchhoff's Laws.

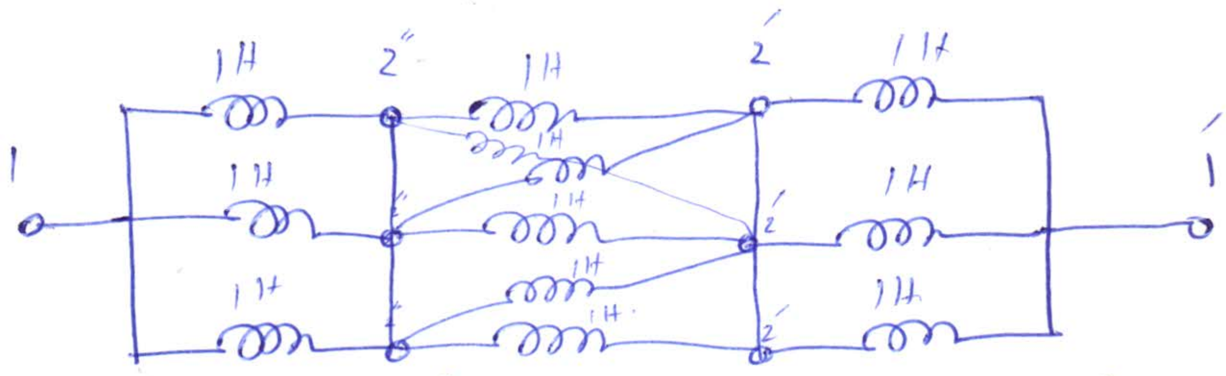


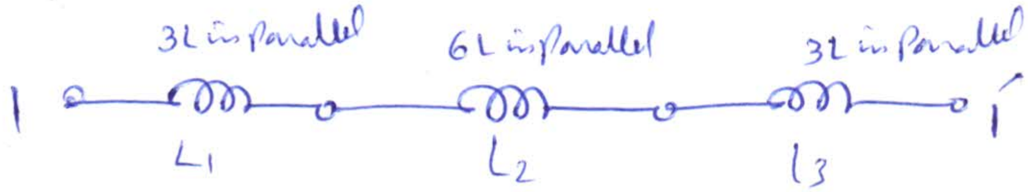
(a)



(b)

Nodes – $2'$ and Nodes – $2''$ are at one Potential as each branch is 1.H., therefore they can be short circuited as below.



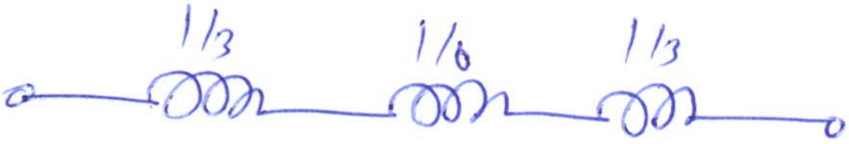


$$\frac{1}{L_1} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$$

$$\frac{1}{L_2} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 6$$

$$\frac{1}{L_3} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1} = 3$$

$$L_1 = \frac{1}{3}, \quad L_2 = \frac{1}{6}, \quad L_3 = \frac{1}{3}.$$

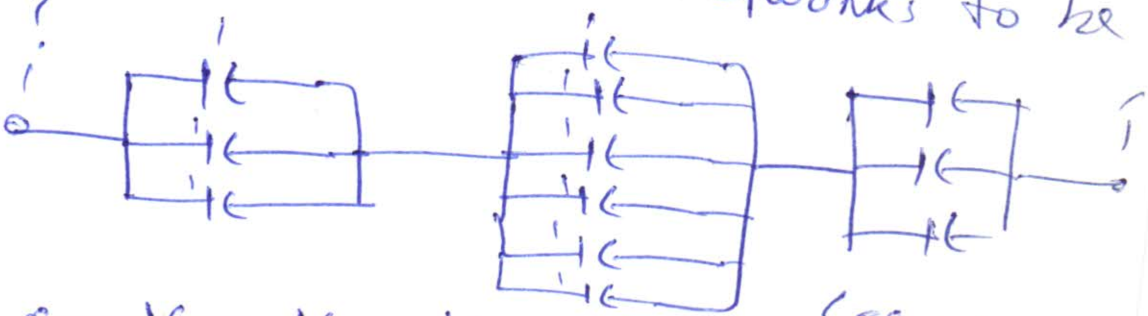


$$L_{eq} = L_1 + L_2 + L_3 = \frac{1}{3} + \frac{1}{6} + \frac{1}{3} = \frac{2+1+2}{6} = \frac{5}{6}\,H$$

$$\boxed{L_{eq} = 5/6\ H}$$

...ob: 3.5 In the network of Prob. 3-4, each 1 H inductor is replaced by 1-Farad capacito... ...d L_{eq} is replaced by C_{eq}. what must be the ...ue of C_{eq} for the two networks to be ...ivalent?

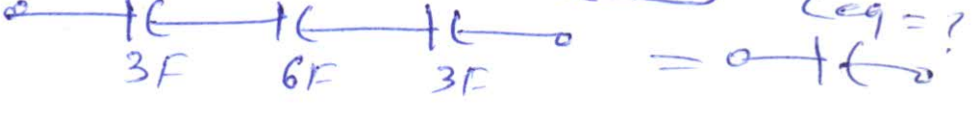
1.



$$... + \frac{1}{6} + \frac{1}{3}.$$

$$\frac{...+1+2}{6} = \frac{5}{6}$$

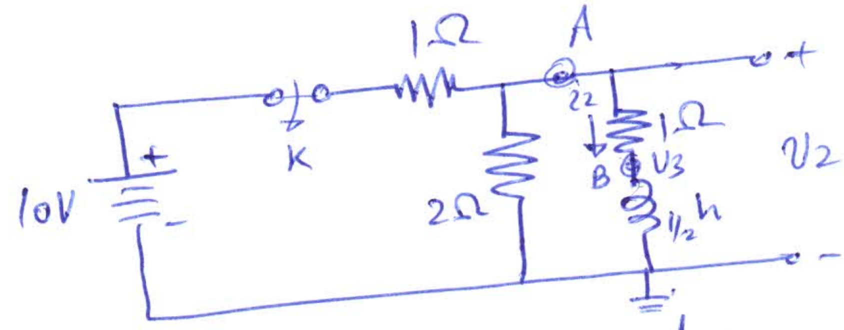
$$\boxed{... = 6/5\,F}$$



Prob: 3.6 This Prob. may be Solved using the two Kirchhoff's laws and Voltage-current relationship for the elements.

At time to after the switch K was closed, it is found that $V_2 = +5V$, You are required to determine the value of $i_2(t_0)$ and $\frac{di_2}{dt}(t_0)$

Sol,



Switch K is closes after time = to.

$V_2(t_0) = 5V.$

Apply K. Current law and write equation at node (A)

$$\frac{V_2 - V_3}{1} + \frac{V_2 - 10}{1} + \frac{V_2}{2} = 0 \longrightarrow (1)$$

Since eq.(1) is valid for all times it is also valid for $t = t_0$, $V_2 = 5V$. So equation (1) becomes

$$\frac{5 - V_3}{1} + \frac{5 - 10}{1} + 5/2 = 0$$

$$V_3(t_0) = 2.5V.$$

$$i_2(t_0) = \frac{V_2(t_0) - V_3(t_0)}{1} = \frac{5 - 2.5}{1} = 2.5 \text{ Ampere.}$$

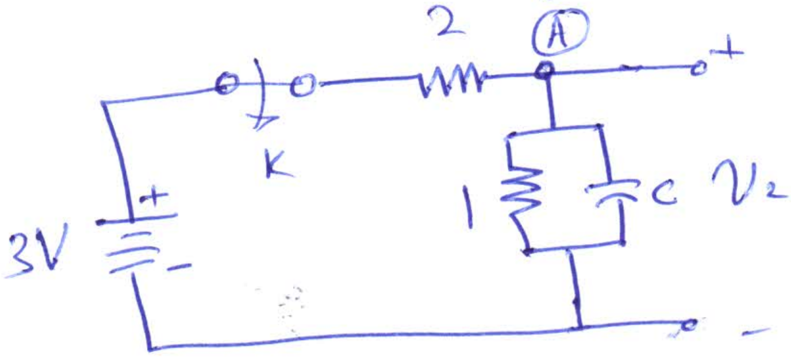
Now also V_3 is the voltage across the inductor, i.e $V_3 = L\frac{di_L}{dt}$ for all times.

$$V_3(t_0) = L\frac{di_L}{dt}(t_0)$$

or

$$\frac{di_L}{dt}(t_0) = \frac{V_3(t_0)}{L} = \frac{2.5}{1/2} = 5\ \text{Amp/sec}$$

Prob: 3.7 This problem is similar to Prob: 3.6. In the network given in the fig, it is given that $V_2(t_0) = 2V$ and $\frac{dV_2}{dt}(t_0) = -10V/\text{sec}$, where t_0 is the time after the switch **K** was closed. Determine the value of "C".



Sol:

Writing Nodal equation at A by Applying K.C.L

$$\frac{V_2 - 0}{1} + \frac{V_2 - 3}{2} + C\frac{dV_2}{dt} = 0 \quad \text{(Valid for all times)}$$

$$\frac{V_2(t_0)}{1} + \frac{V_2(t_0) - 3}{2} + C\frac{dV_2}{dt}(t_0) = 0$$

Now putting the given values

$$\frac{2}{1}+\frac{2-3}{2}+C\times(-10)=0$$

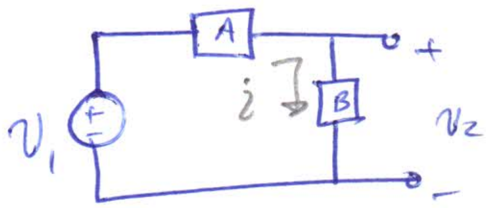
$$2-\frac{1}{2}-10C=0$$

or

$$10C=\frac{3}{2} \quad \text{or} \quad C=\frac{3}{2\times 10}=0.15\,F$$

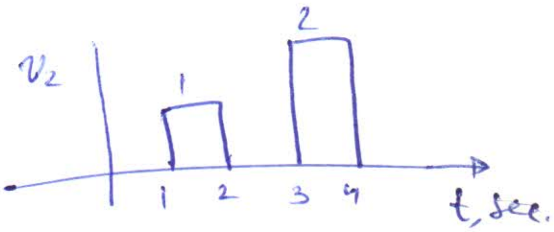
$$\boxed{C=0.15 \text{ Farads}}$$

Prob: 3.8 onwards:

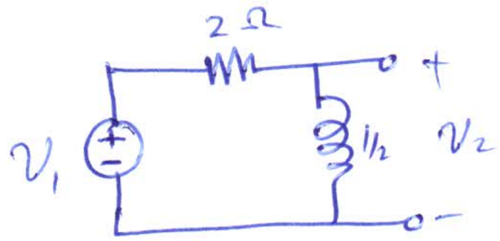


$A = R = 2$

$B = L = 1/2.$



$v_2 =$ a, b, c, d, e, f.



Initial Conditions = 0.

$$\boxed{\text{Find \& Draw } v_1 = ?}$$

$$v_1 = i(t).R + v_2.$$

But $v_2 = L\dfrac{di(t)}{dt}$

So

$$v_1 = i(t).R + L\frac{d}{dt}i(t).$$

or

$$\boxed{v_1 = 2.\,i(t) + \frac{1}{2}\frac{di(t)}{dt}} \longleftrightarrow (1)$$