3: Nodal Analysis

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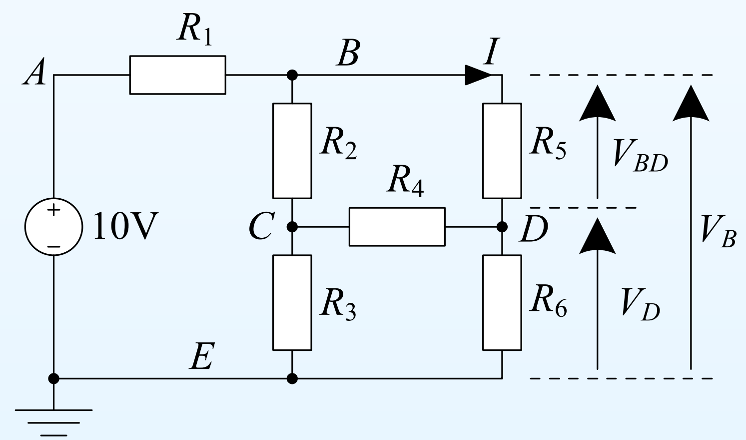
Reminders:

A node is all the points in a circuit that are directly interconnected. We assume the interconnections have zero resistance so all points within a node have the same voltage. Five nodes: $A, \cdots, E$.



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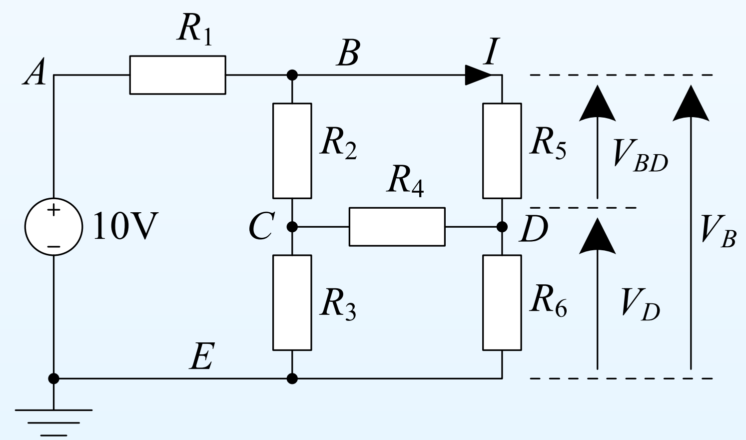

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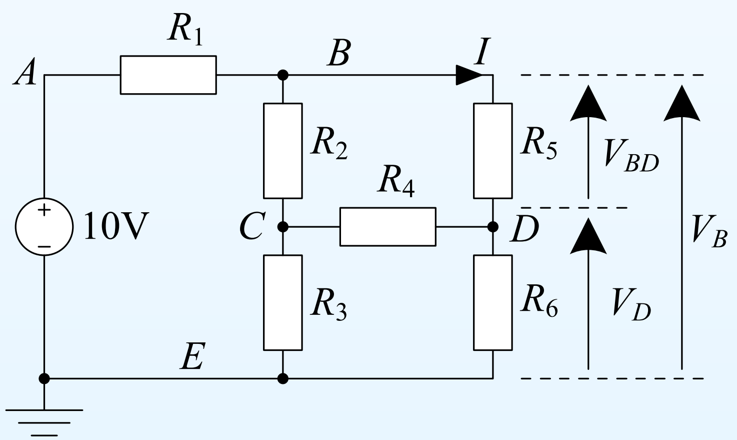

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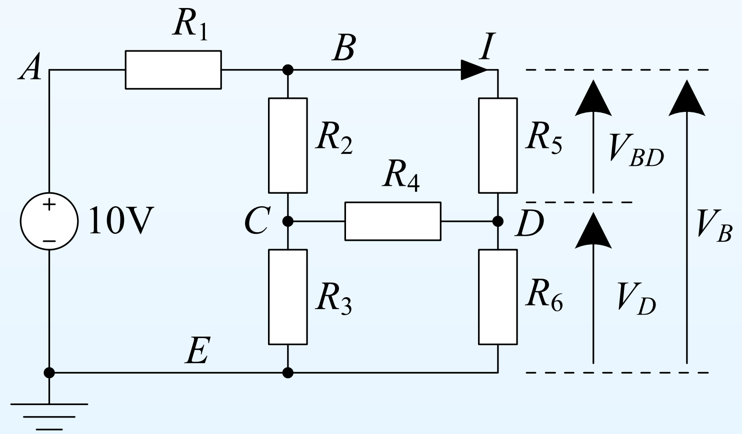

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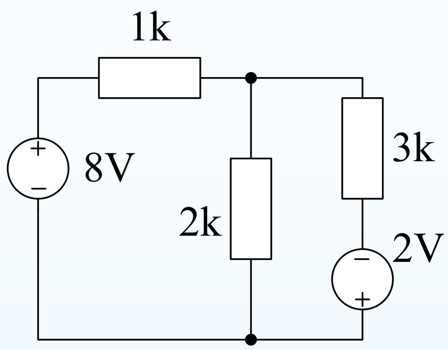
KVL: $V_{BD} = V_B - V_D$

KCL: Total current exiting any closed region is zero.

Nodal Analysis Stage 1: Label Nodes

To find the voltage at each node, the first step is to label each node with its voltage as follows



(1) Pick any node as the voltage reference. Label its voltage as 0 V.
(2) If any fixed voltage sources are connected to a labelled node, label their other ends by adding the value of the source onto the voltage of the labelled end.
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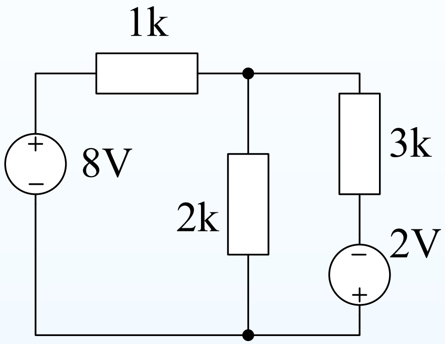

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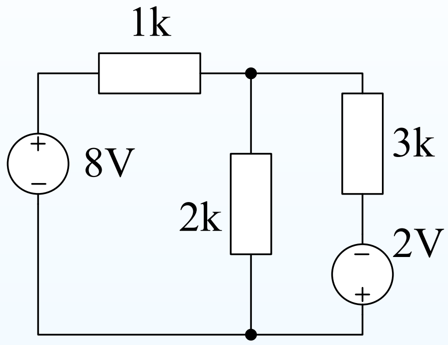

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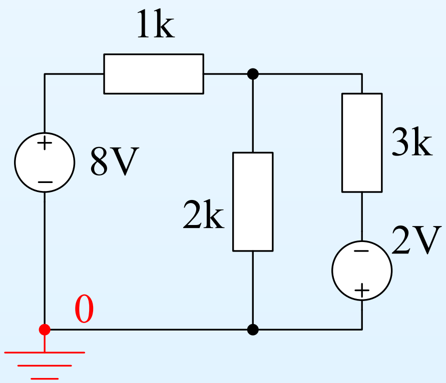

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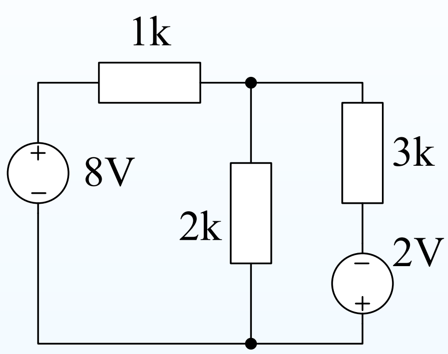

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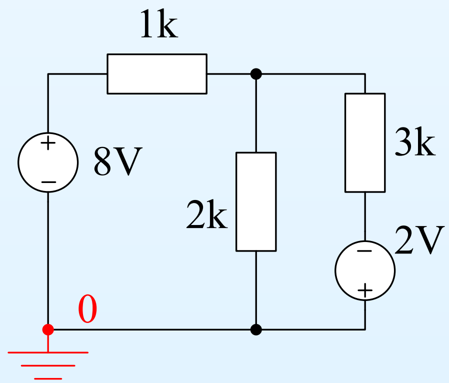

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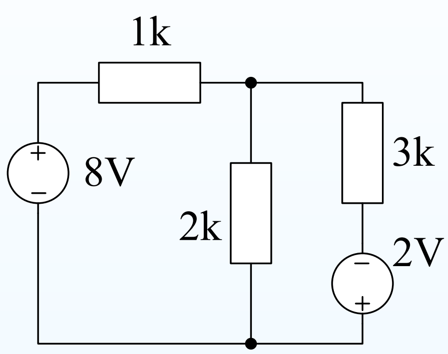

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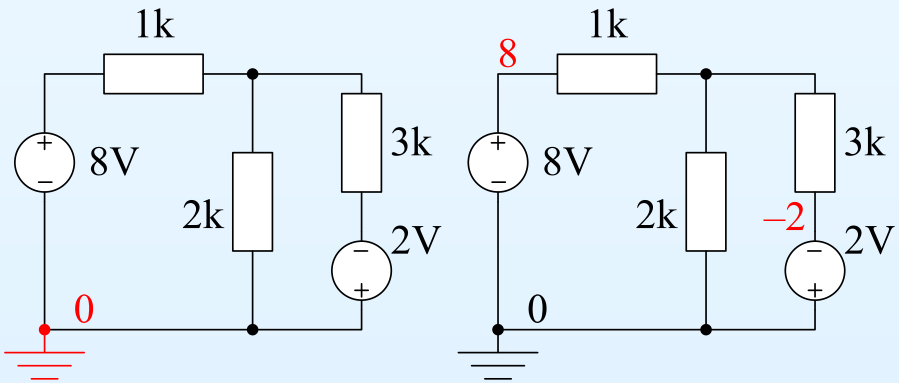

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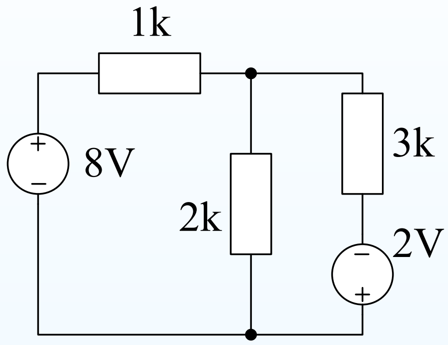

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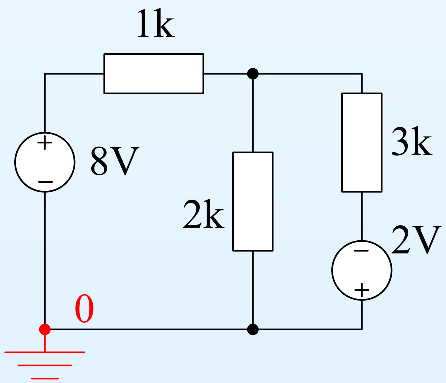

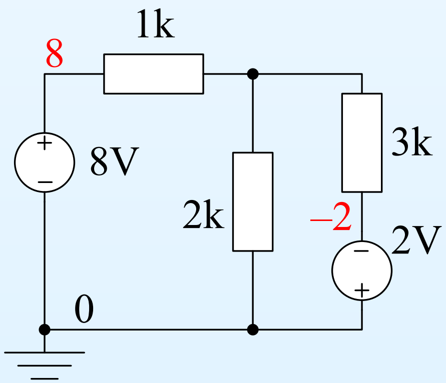

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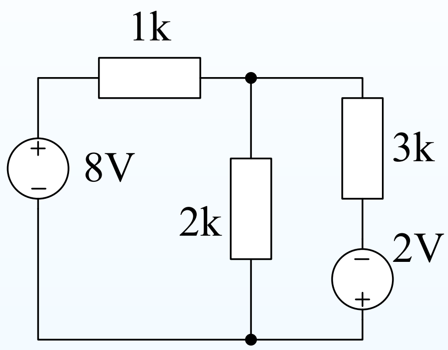

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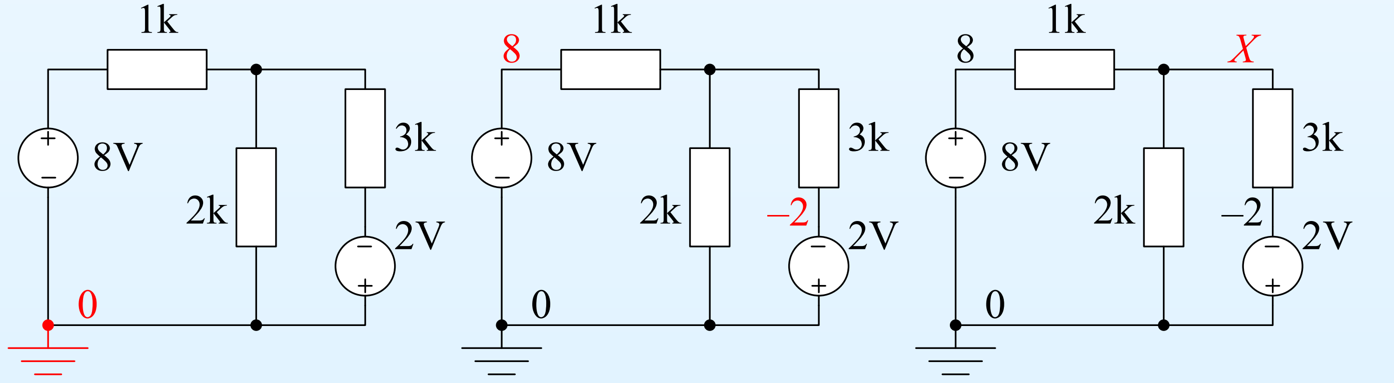
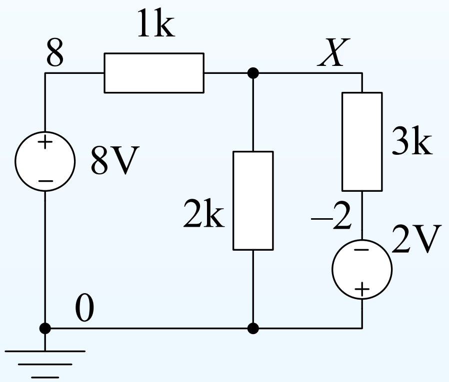


Nodal Analysis Stage 2: KCL Equations

The second step is to write down a KCL equation for each node labelled with a variable by setting the total current flowing out of the node to zero. For a circuit with N nodes and S voltage sources you will have $N - S - 1$ simultaneous equations to solve.



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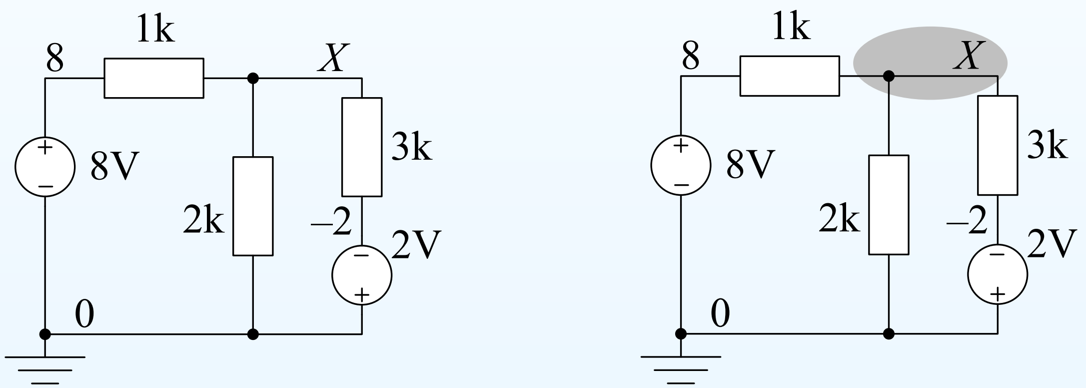

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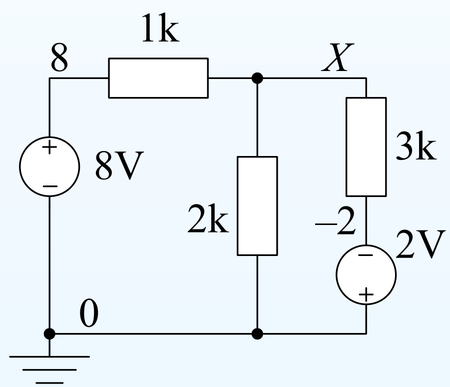

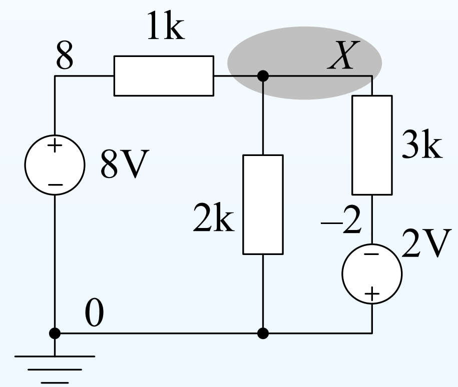

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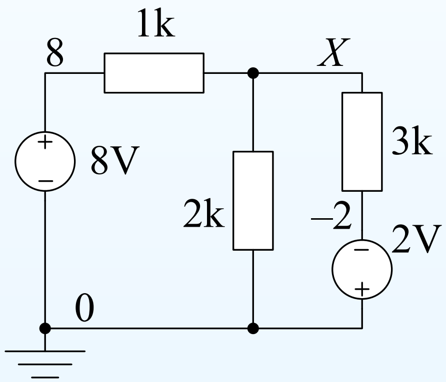

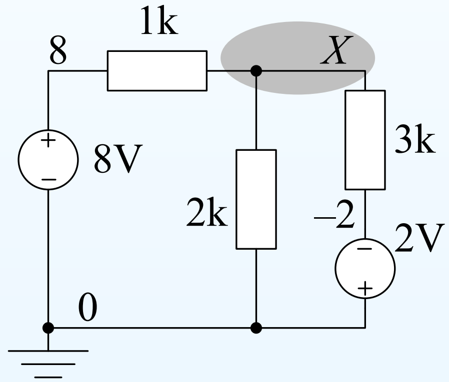

We only have one variable:

$$\frac{X-8}{1\text{ k}} + \frac{X-0}{2\text{ k}} + \frac{X-(-2)}{3\text{ k}} = 0 \quad \Rightarrow \quad (6X - 48) + 3X + (2X + 4) = 0$$

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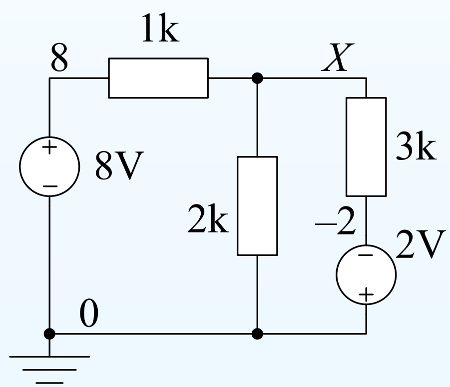

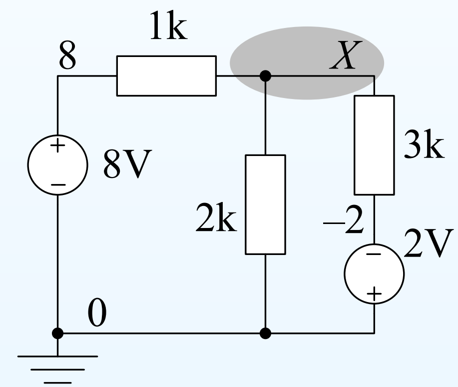

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$$11X = 44 \quad \Rightarrow \quad X = 4$$

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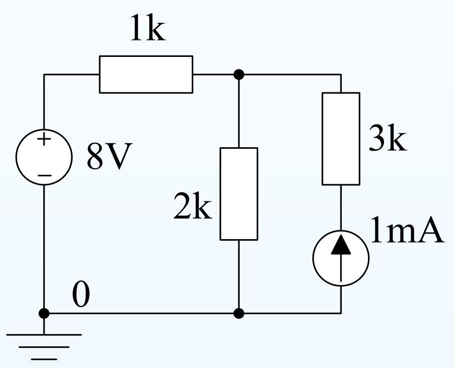

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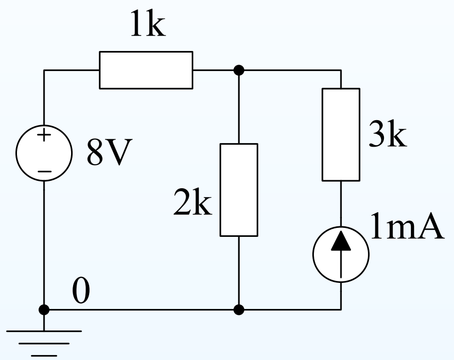

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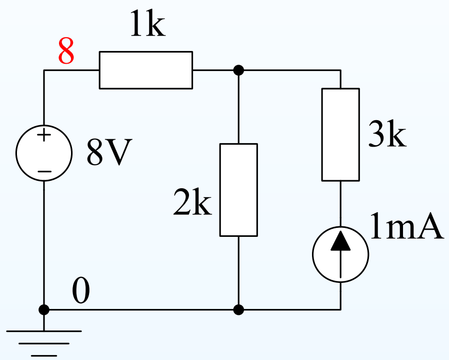
(2) Label nodes: 8



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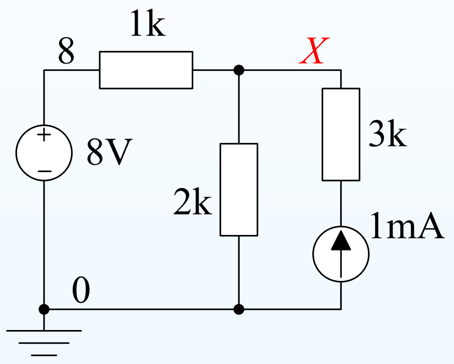

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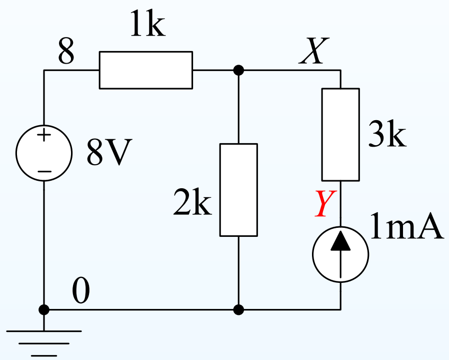

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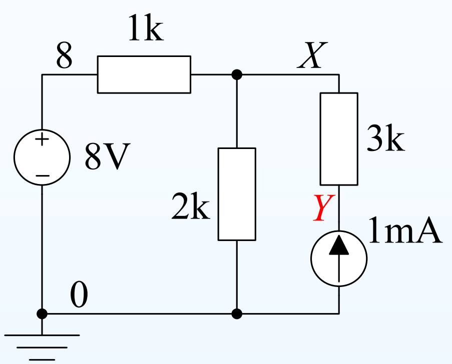
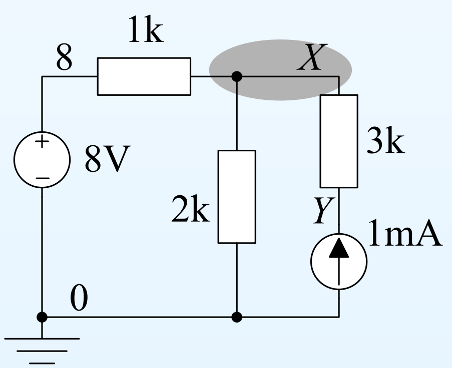


(3) Write equations

$$\frac{X-8}{1} + \frac{X}{2} + \frac{X-Y}{3} = 0$$



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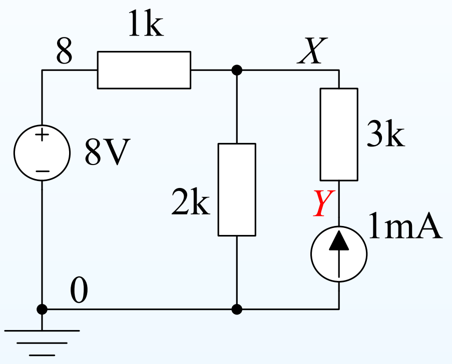

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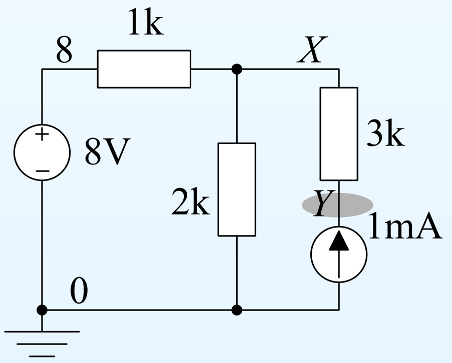
$$\frac{Y-X}{3} + (-1) = 0$$



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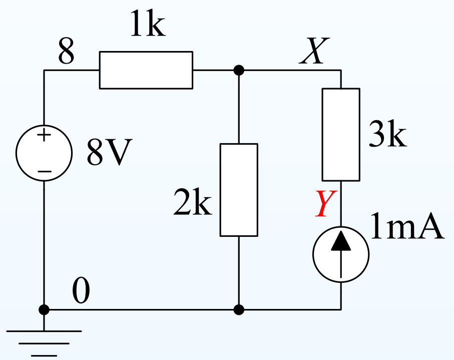

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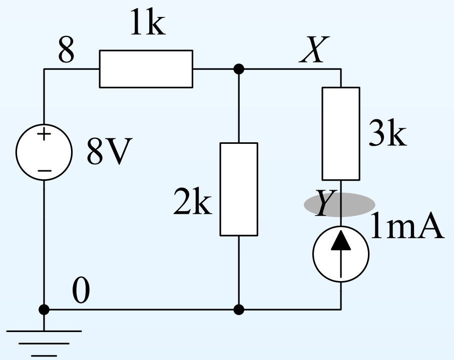

Ohm's law works OK if all resistors are in $\mathrm{k}\Omega$ and all currents in mA.

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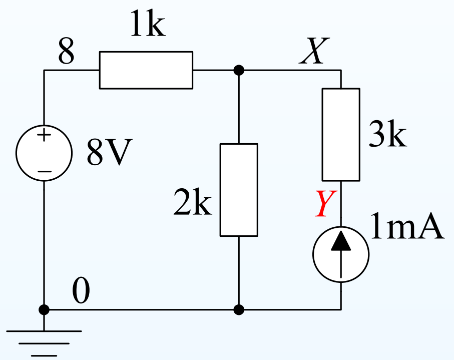

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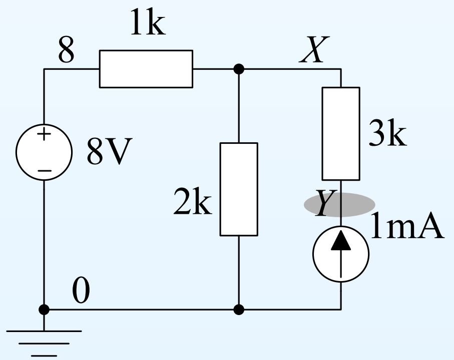

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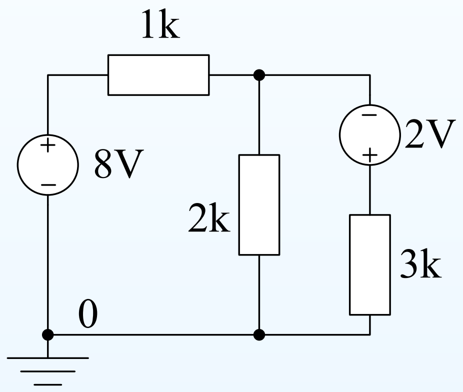
(4) Solve the equations: $X=6,\ Y=9$

Floating Voltage Sources

Floating voltage sources have neither end connected to a known fixed voltage. We have to change how we form the KCL equations slightly.



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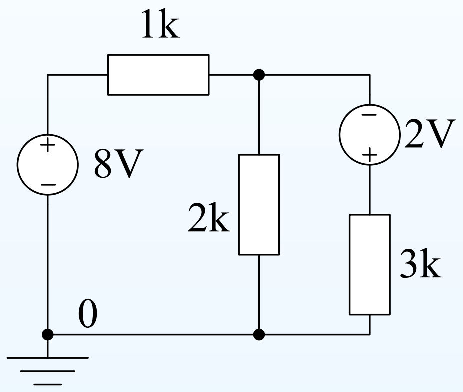

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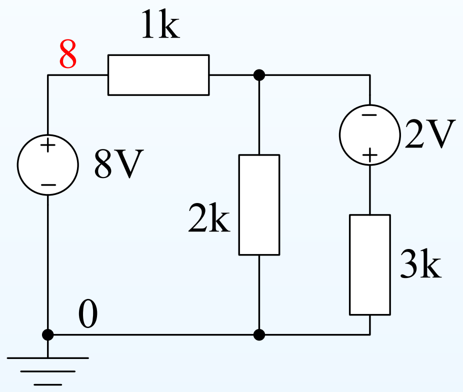

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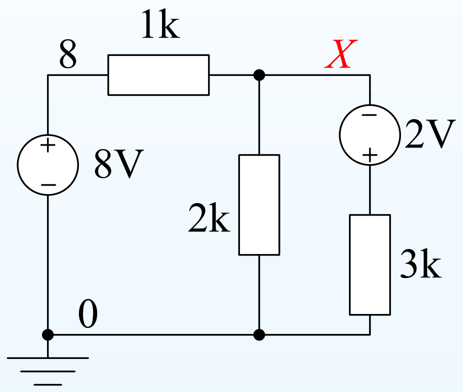

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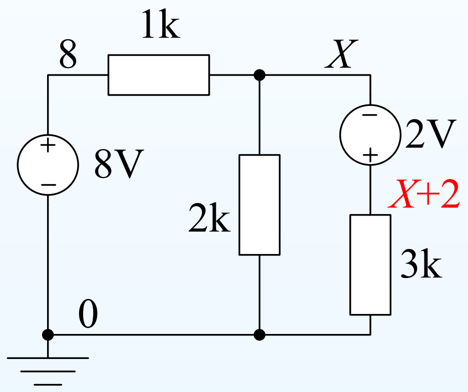

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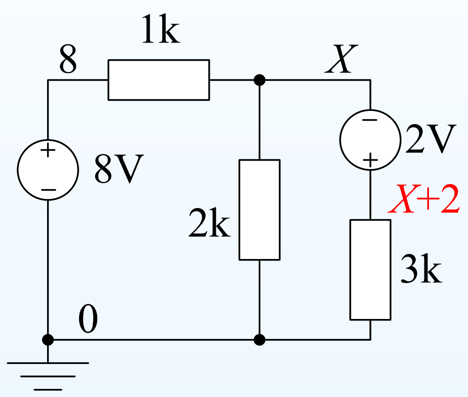
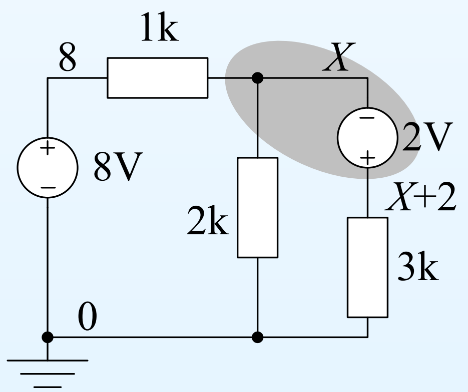
(3) Write KCL equations but count all the nodes connected via floating voltage sources as a single "super-node" giving one equation

$$\frac{X-8}{1}+\frac{X}{2}+\frac{(X+2)-0}{3}=0$$





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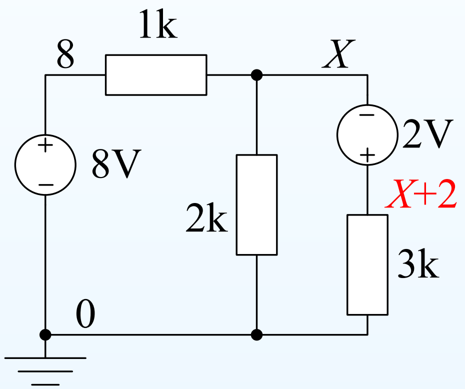

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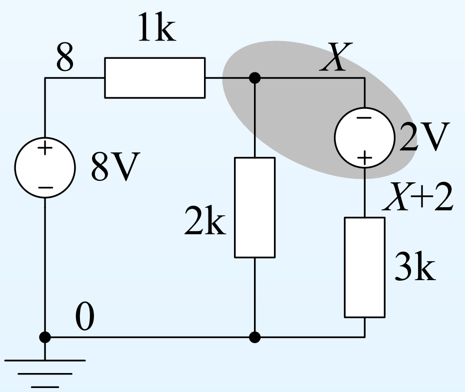

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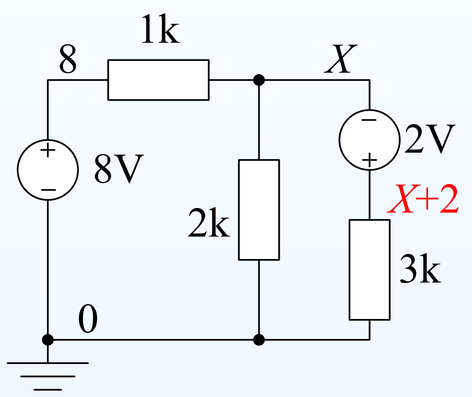

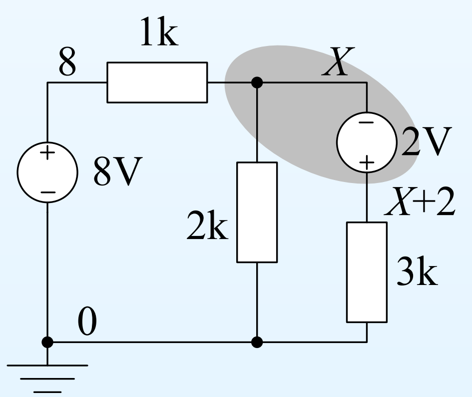

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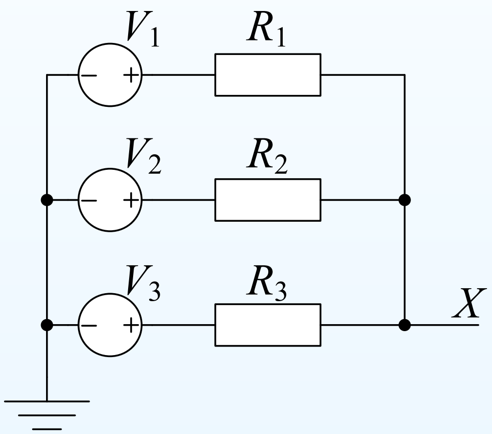

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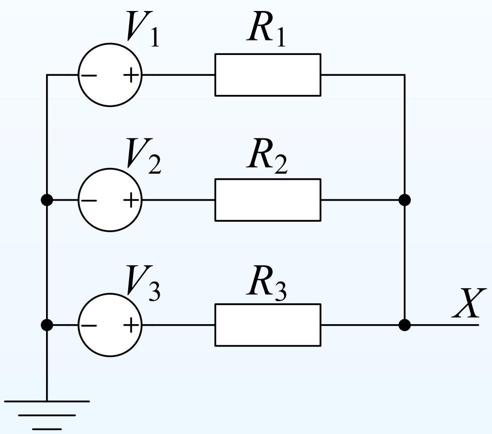

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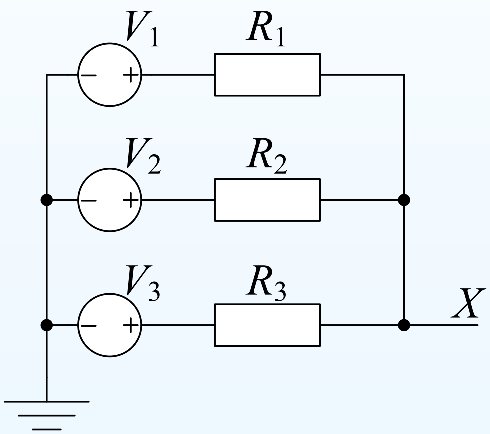

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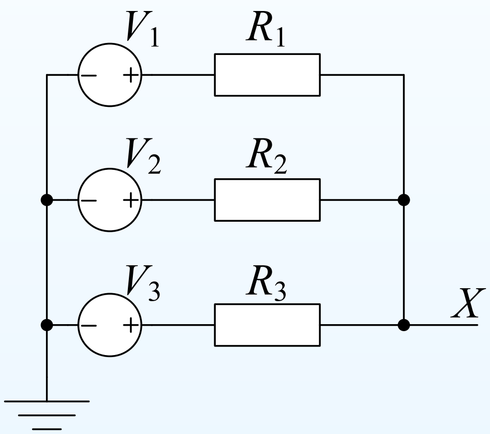

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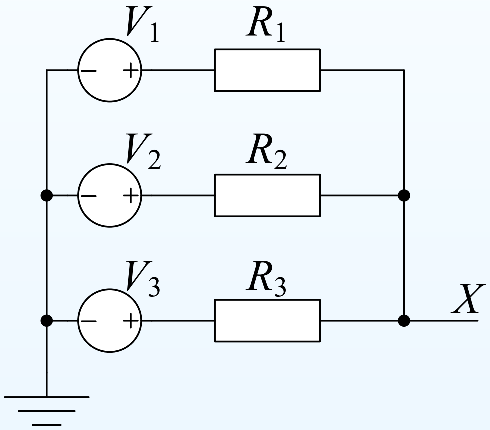

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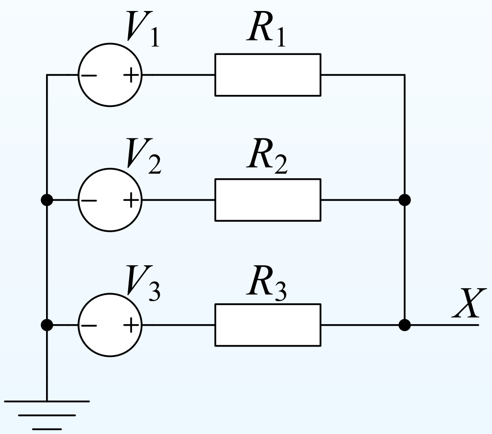

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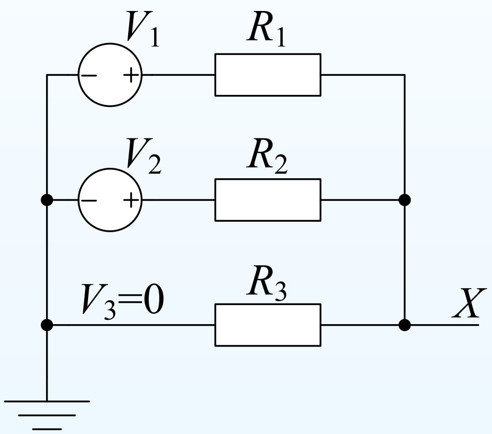

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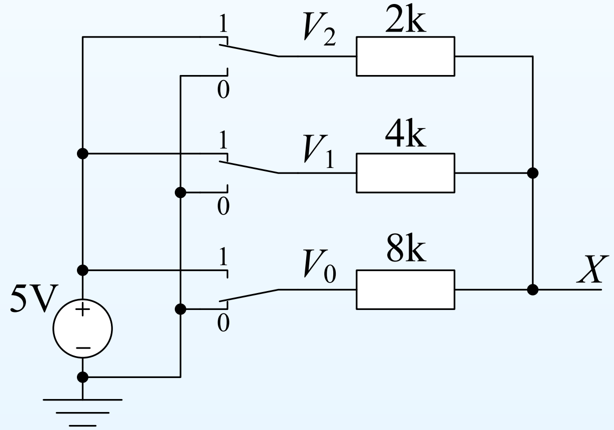

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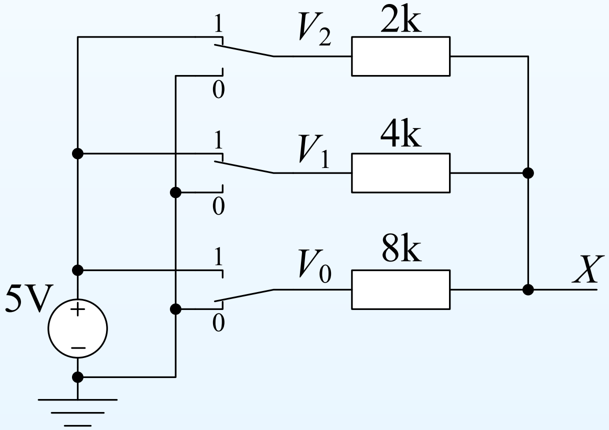

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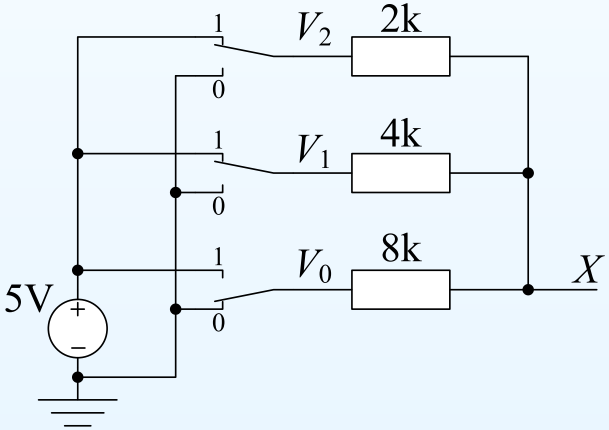

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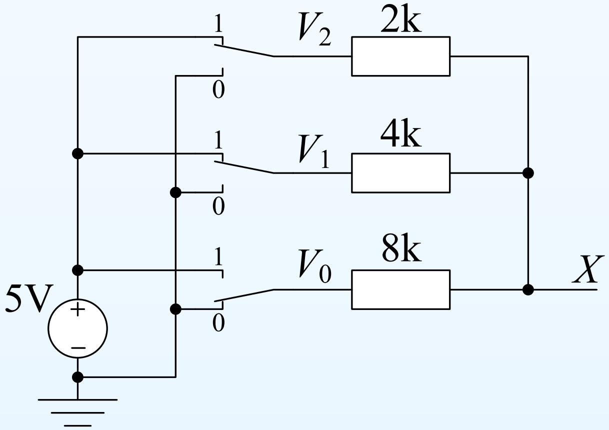

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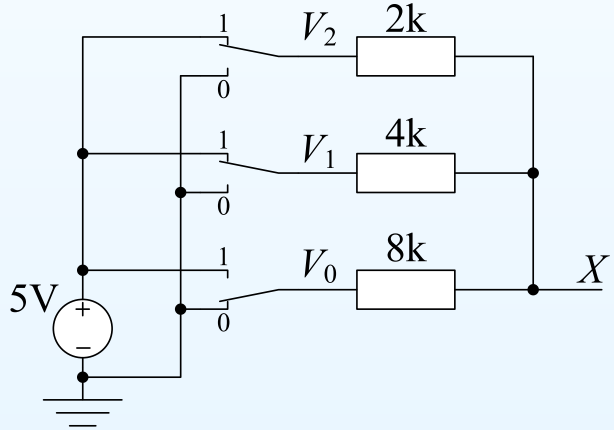

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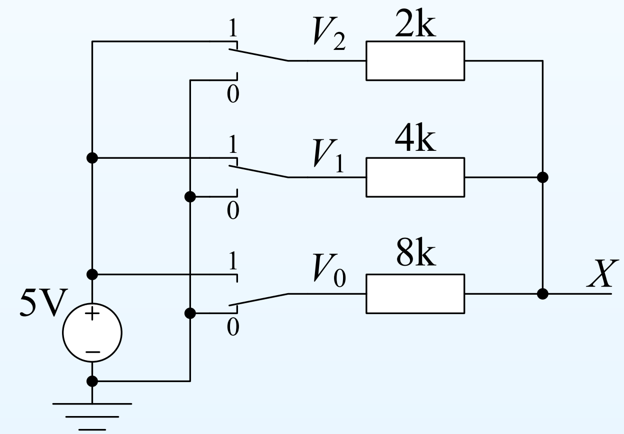

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So we have made a circuit in which X is proportional to a binary number b.

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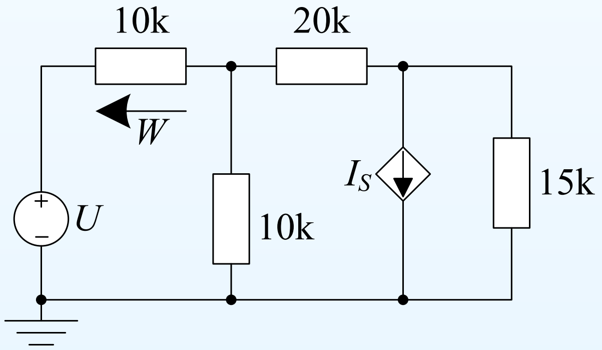

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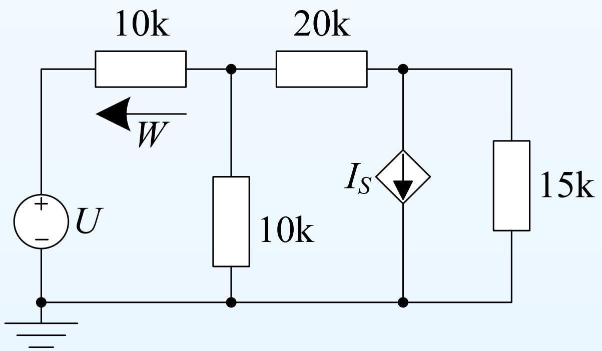

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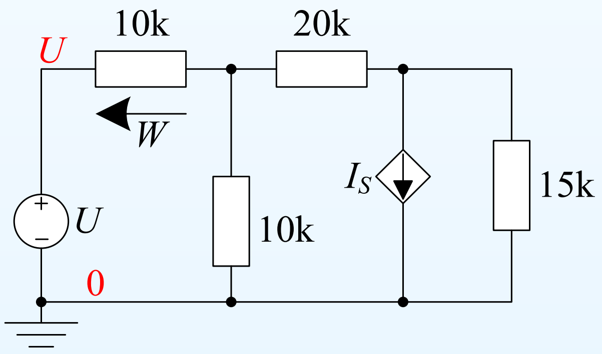

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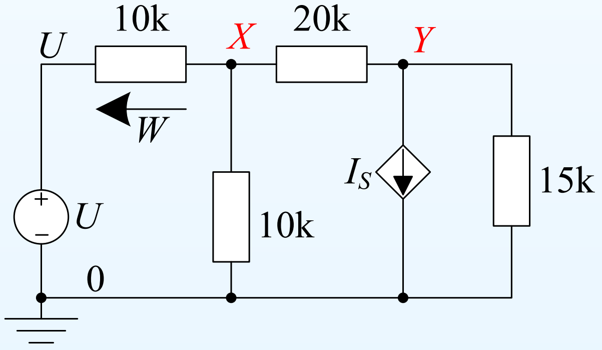

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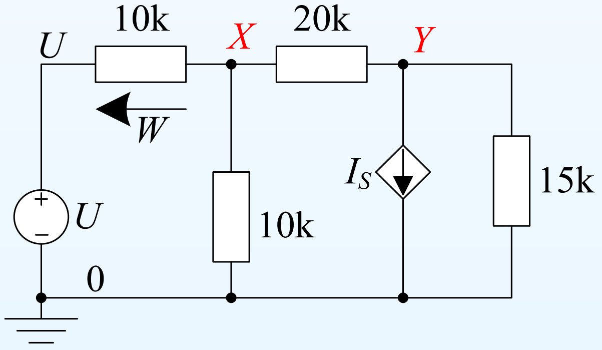

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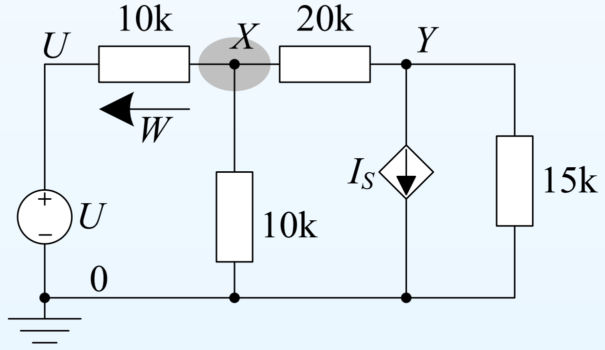

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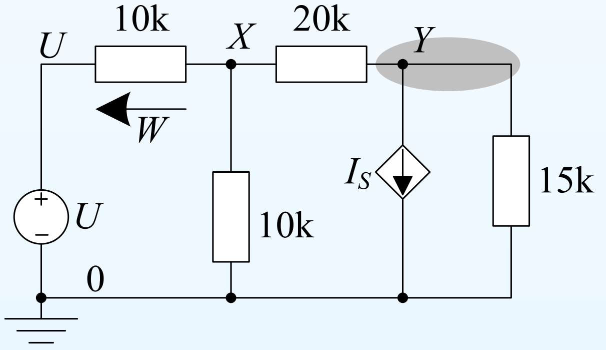

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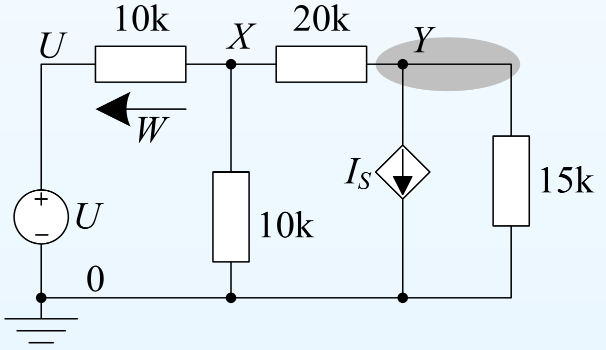

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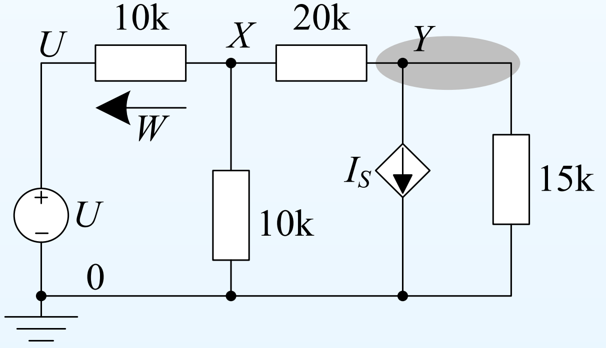

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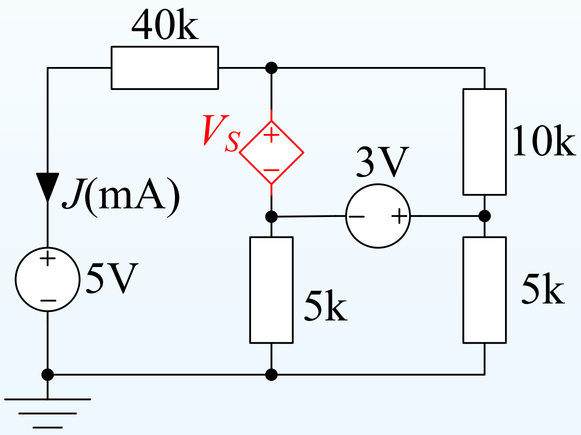

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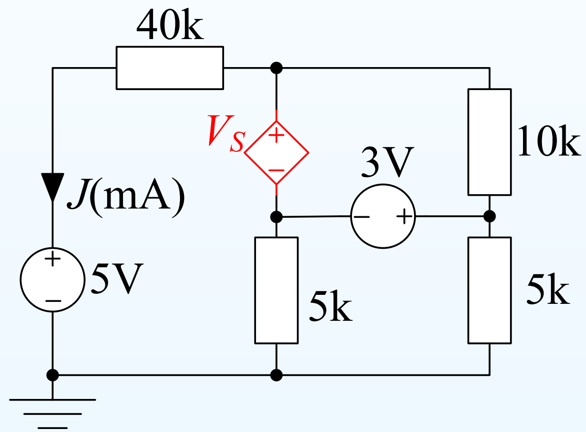

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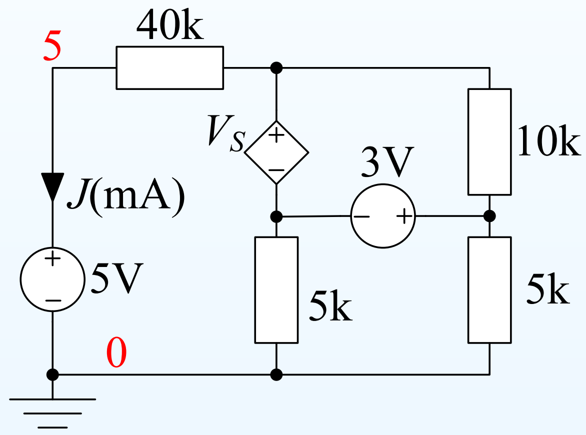

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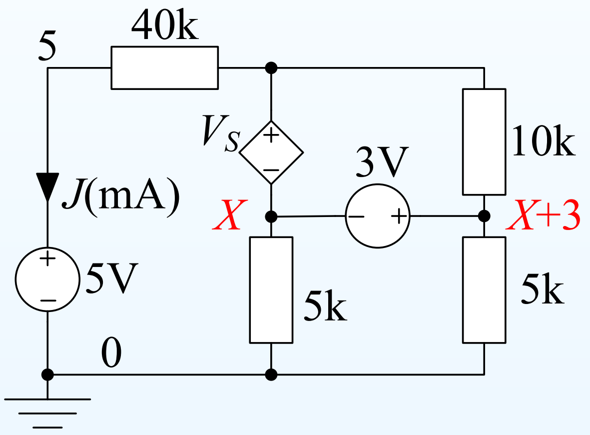

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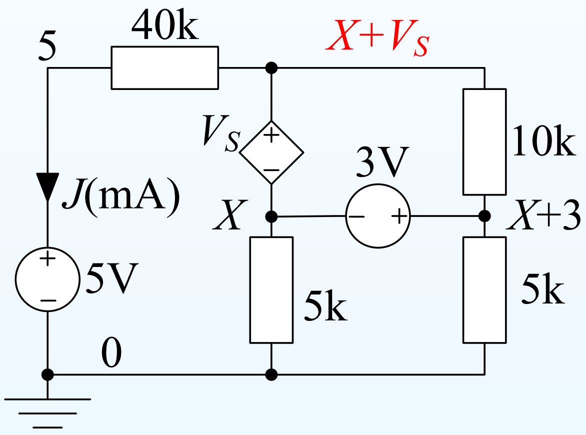

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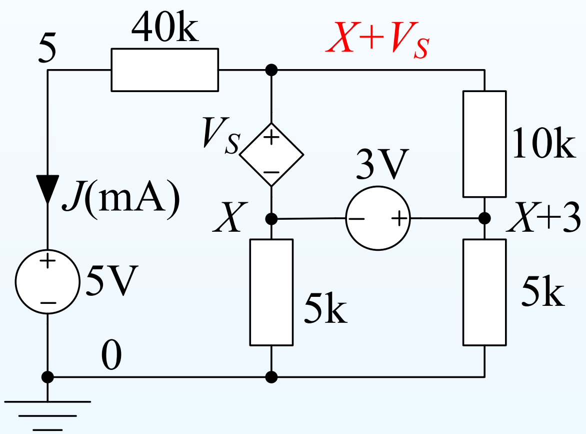

$$V_S = 10J = 10 \times \frac{X+V_S-5}{40} \Rightarrow 3V_S = X - 5$$

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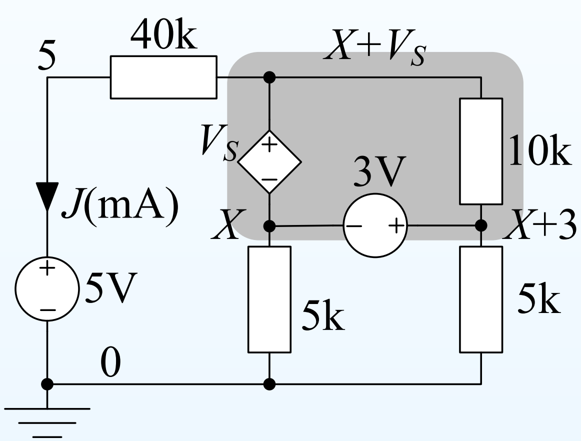

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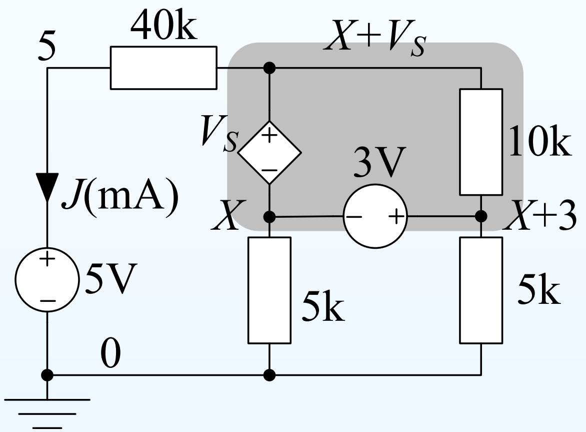

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(1) Pick any node as the voltage reference. Label its voltage as 0 V. Label any dependent sources with $V_S,\ I_S,\ \ldots$.

(2) If any voltage sources are connected to a labelled node, label their other ends by adding the value of the source onto the voltage of the labelled end.

(3) Pick an unlabelled node and label it with $X,\ Y,\ \ldots$, then loop back to step (2) until all nodes are labelled.

(4) For each **dependent source**, write down an equation that expresses its value in terms of other node voltages.

(5) Write down a KCL equation for each “normal” node (i.e. one that is not connected to a floating voltage source).

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