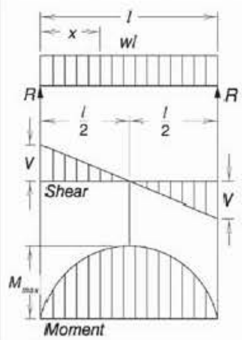
Table 3-23
Shears, Moments and Deflections

1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

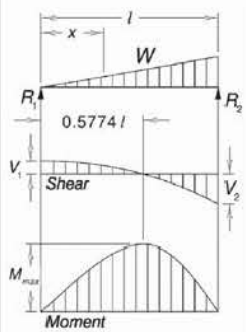
M_{max} (at center) .... $= \frac{wl^2}{8}$

M_x .... $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$

Δ_x .... $= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

2. SIMPLE BEAM — LOAD INCREASING UNIFORMLY TO ONE END



Total Equiv. Uniform Load .... $= \frac{16W}{9\sqrt{3}} = 1.03\,W$

$R_1 = V_1$ .... $= \frac{W}{3}$

$R_2 = V_2 = V_{max}$ .... $= \frac{2W}{3}$

V_x .... $= \frac{W}{3} - \frac{Wx^2}{l^2}$

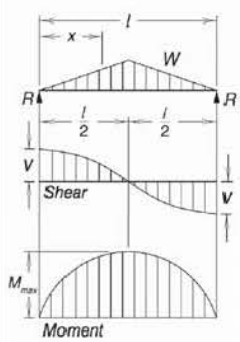
M_{max} $\left(\text{at } x = \frac{l}{\sqrt{3}} = 0.557\,l\right)$ .... $= \frac{2Wl}{9\sqrt{3}} = 0.128\,Wl$

M_x .... $= \frac{Wx}{3l^2}(l^2 - x^2)$

Δ_{max} $\left(\text{at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = 0.519\,l\right)$ .... $= 0.0130\frac{Wl^3}{EI}$

Δ_x .... $= \frac{Wx}{180EIl^2}\left(3x^4 - 10l^2x^2 + 7l^4\right)$

3. SIMPLE BEAM — LOAD INCREASING UNIFORMLY TO CENTER



Total Equiv. Uniform Load .... $= \frac{4W}{3}$

$R = V$ .... $= \frac{W}{2}$

V_x $\left(\text{when } x < \frac{l}{2}\right)$ .... $= \frac{W}{2l^2}\left(l^2 - 4x^2\right)$

M_{max} (at center) .... $= \frac{Wl}{6}$

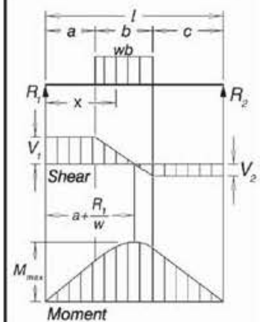
M_x $\left(\text{when } x < \frac{l}{2}\right)$ .... $= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$

Δ_{max} (at center) .... $= \frac{Wl^3}{60EI}$

Δ_x $\left(\text{when } x < \frac{l}{2}\right)$ .... $= \frac{Wx}{480EIl^2}\left(5l^2 - 4x^2\right)^2$

Table 3-23 (continued)
Shears, Moments and Deflections

4. SIMPLE BEAM — UNIFORM LOAD PARTIALLY DISTRIBUTED



$R_1 = V_1$ (max. when $a < c$) .... $= \frac{wb}{2l}(2c+b)$

$R_2 = V_2$ (max. when $a > c$) .... $= \frac{wb}{2l}(2a+b)$

V_x (when $x > a$ and $< (a+b)$) .... $= R_1 - w(x-a)$

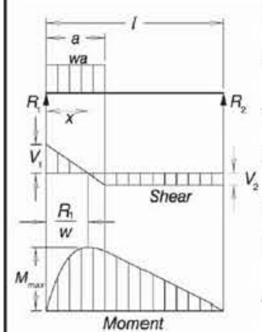
M_{max} $\left(\text{at } x = a + \frac{R_1}{w}\right)$ .... $= R_1\left(a + \frac{R_1}{2w}\right)$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $x > a$ and $< (a+b)$) .... $= R_1 x - \frac{w}{2}(x-a)^2$

M_x (when $x > (a+b)$) .... $= R_2(l-x)$

5. SIMPLE BEAM — UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END



$R_1 = V_1 = V_{max}$ .... $= \frac{wa}{2l}(2l-a)$

$R_2 = V_2$ .... $= \frac{wa^2}{2l}$

V_x (when $x < a$) .... $= R_1 - wx$

M_{max} $\left(\text{at } x = \frac{R_1}{w}\right)$ .... $= \frac{R_1^2}{2w}$

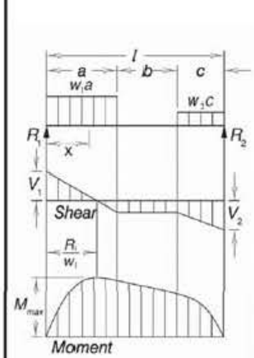
M_x (when $x < a$) .... $= R_1 x - \frac{wx^2}{2}$

M_x (when $x > a$) .... $= R_2(l-x)$

Δ_x (when $x < a$) .... $= \frac{wx}{24EIl}\left(a^2(2l-a)^2 - 2ax^2(2l-a) + lx^3\right)$

Δ_x (when $x > a$) .... $= \frac{wa^2(l-x)}{24EIl}\left(4xl - 2x^2 - a^2\right)$

6. SIMPLE BEAM — UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END



$R_1 = V_1$ .... $= \frac{w_1 a(2l-a) + w_2 c^2}{2l}$

$R_2 = V_2$ .... $= \frac{w_2 c(2l-c) + w_1 a^2}{2l}$

V_x (when $x < a$) .... $= R_1 - w_1 x$

V_x (when $a < x < (a+b)$) .... $= R_1 - w_1 a$

V_x (when $x > (a+b)$) .... $= R_2 - w_2(l-x)$

M_{max} $\left(\text{at } x = \frac{R_1}{w_1}, \text{ when } R_1 < w_1 a\right)$ .... $= \frac{R_1^2}{2w_1}$

M_{max} $\left(\text{at } x = l - \frac{R_2}{w_2}, \text{ when } R_2 < w_2 c\right)$ $= \frac{R_2^2}{2w_2}$

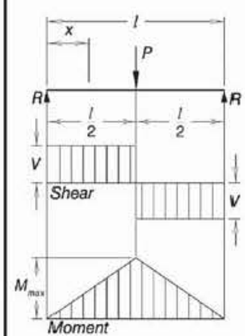
M_x (when $x < a$) .... $= R_1 x - \frac{w_1 x^2}{2}$

M_x (when $a < x < (a+b)$) .... $= R_1 x - \frac{w_1 a}{2}(2x-a)$

M_x (when $x > (a+b)$) .... $= R_2(l-x) - \frac{w_2(l-x)^2}{2}$

Table 3-23 (continued)
Shears, Moments and Deflections

7. SIMPLE BEAM — CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load .... $= 2P$

$R = V$ .... $= \frac{P}{2}$

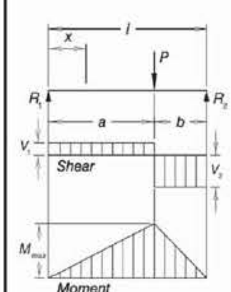
M_{max} (at point of load) .... $= \frac{Pl}{4}$

M_x (when $x < \frac{l}{2}$) .... $= \frac{Px}{2}$

Δ_{max} (at point of load) .... $= \frac{Pl^3}{48EI}$

Δ_x (when $x < \frac{l}{2}$) .... $= \frac{Px}{48EI}\left(3l^2 - 4x^2\right)$

8. SIMPLE BEAM — CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load .... $= \frac{8Pab}{l^2}$

$R_1 = V_1$ ($= V_{max}$ when $a < b$) .... $= \frac{Pb}{l}$

$R_2 = V_2$ ($= V_{max}$ when $a > b$) .... $= \frac{Pa}{l}$

M_{max} (at point of load) .... $= \frac{Pab}{l}$

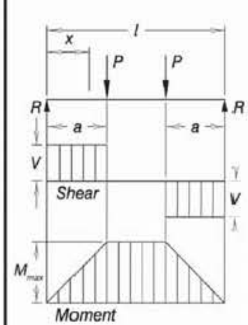
M_x (when $x < a$) .... $= \frac{Pbx}{l}$

Δ_{max} $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}}, \text{ when } a > b\right)$ .... $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$

Δ_a (at point of load) .... $= \frac{Pa^2b^2}{3EIl}$

Δ_x (when $x < a$) .... $= \frac{Pbx}{6EIl}\left(l^2 - b^2 - x^2\right)$

9. SIMPLE BEAM — TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED



Total Equiv. Uniform Load .... $= \frac{8Pa}{l}$

$R = V$ .... $= P$

M_{max} (between loads) .... $= Pa$

M_x (when $x < a$) .... $= Px$

Δ_{max} (at center) .... $= \frac{Pa}{24EI}\left(3l^2 - 4a^2\right)$

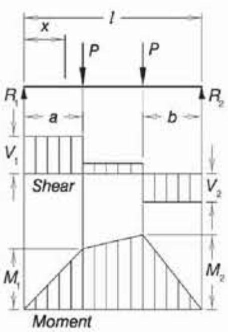
Δ_{max} (when $a = \frac{l}{3}$) .... $= \frac{Pl^3}{28EI}$

Δ_x (when $x < a$) .... $= \frac{Px}{6EI}\left(3la - 3a^2 - x^2\right)$

Δ_x (when $a < x < (l - a)$) .... $= \frac{Pa}{6EI}\left(3lx - 3x^2 - a^2\right)$

Table 3-23 (continued)
Shears, Moments and Deflections

10. SIMPLE BEAM — TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ ($= V_{max}$ when $a < b$) .... $= \frac{P}{l}(l - a + b)$

$R_2 = V_2$ ($= V_{max}$ when $a > b$) .... $= \frac{P}{l}(l - b + a)$

V_x (when $a < x < (l - b)$) .... $= \frac{P}{l}(b - a)$

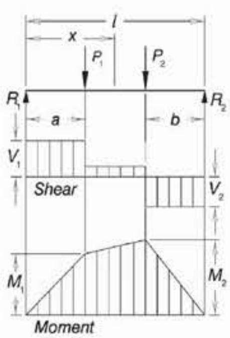
M_1 ($= M_{max}$ when $a > b$) .... $= R_1 a$

M_2 ($= M_{max}$ when $a < b$) .... $= R_2 b$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $a < x < (l - b)$) .... $= R_1 x - P(x - a)$

11. SIMPLE BEAM — TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED



$R_1 = V_1$ .... $= \frac{P_1(l - a) + P_2 b}{l}$

$R_2 = V_2$ .... $= \frac{P_1 a + P_2(l - b)}{l}$

V_x (when $a < x < (l - b)$) .... $= R_1 - P_1$

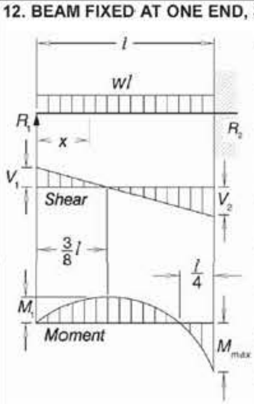
M_1 ($= M_{max}$ when $R_1 < P_1$) .... $= R_1 a$

M_2 ($= M_{max}$ when $R_2 < P_2$) .... $= R_2 b$

M_x (when $x < a$) .... $= R_1 x$

M_x (when $a < x < (l - b)$) .... $= R_1 x - P_1(x - a)$

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER — UNIFORMLY DISTRUBTED LOAD



Total Equiv. Uniform Load .... $= wl$

$R_1 = V_1$ .... $= \frac{3wl}{8}$

$R_2 = V_2 = V_{max}$ .... $= \frac{5wl}{8}$

V_x .... $= R_1 - wx$

M_{max} .... $= \frac{wl^2}{8}$

M_1 (at $x = \frac{3}{8}l$) .... $= \frac{9}{128}wl^2$

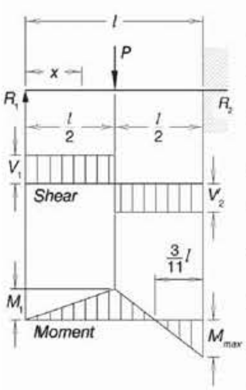
M_x .... $= R_1 x - \frac{wx^2}{2}$

Δ_{max} (at $x = \frac{l}{16}(1 + \sqrt{33}) = 0.422\,l$) .... $= \frac{wl^4}{185\,EI}$

Δ_x .... $= \frac{wx}{48\,EI}(l^3 - 3lx^2 + 2x^3)$

Table 3-23 (continued)
Shears, Moments and Deflections

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER — CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load .... $= \frac{3P}{2}$

$R_1 = V_1$ .... $= \frac{5P}{16}$

$R_2 = V_2 = V_{max}$ .... $= \frac{11P}{16}$

M_{max} (at fixed end) .... $= \frac{3Pl}{16}$

M_1 (at point of load) .... $= \frac{5Pl}{32}$

M_x (at $x < \frac{l}{2}$) .... $= \frac{5Px}{16}$

M_x (when $x > \frac{l}{2}$) .... $= P\left(\frac{l}{2} - \frac{11x}{16}\right)$

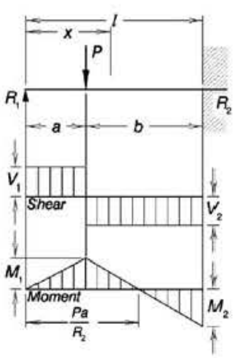
Δ_{max} (at $x = \frac{l}{\sqrt{5}} = 0.447\,l$) .... $= \frac{Pl^3}{48EI\sqrt{5}} = 0.00932\frac{Pl^3}{EI}$

Δ_x (at point of load) .... $= \frac{7Pl^3}{768EI}$

Δ_x (at $x < \frac{l}{2}$) .... $= \frac{Px}{96EI}\left(3l^2 - 5x^2\right)$

Δ_x (at $x > \frac{l}{2}$) .... $= \frac{P}{96EI}(x-l)^2(11x - 2l)$

14. BEAM FIXED AT ONE END, SUPPORTED AT THE OTHER — CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ .... $= \frac{Pb^2}{2l^3}(a + 2l)$

$R_2 = V_2$ .... $= \frac{Pa}{2l^3}\left(3l^2 - a^2\right)$

M_1 (at point of load) .... $= R_1 a$

M_2 (at fixed end) .... $= \frac{Pab}{2l^2}(a + l)$

M_x (at $x < a$) .... $= R_1 x$

M_x (when $x > a$) .... $= R_1 x - P(x - a)$

Δ_{max} $\left(\text{when } a < 0.414l \text{ at } x = l\frac{\left(l^2 + a^2\right)}{\left(3l^2 - a^2\right)}\right)$ .. $= \frac{Pa}{3EI}\frac{\left(l^2 - a^2\right)^3}{\left(3l^2 - a^2\right)^2}$

Δ_{max} $\left(\text{when } a > 0.414l \text{ at } x = l\sqrt{\frac{a}{2l + a}}\right)$ ..... $= \frac{Pab^2}{6EI}\sqrt{\frac{a}{2l + a}}$

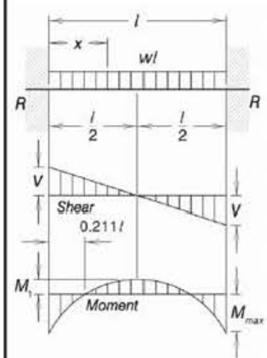
Δ_a (at point of load) .... $= \frac{Pa^2b^3}{12EIl^3}(3l + a)$

Δ_x (when $x < a$) .... $= \frac{Pb^2x}{12EIl^3}\left(3al^2 - 2lx^2 - ax^2\right)$

Δ_x (when $x > a$) .... $= \frac{Pa}{12EIl^3}(l - x)^2\left(3l^2x - a^2x - 2a^2l\right)$

Table 3-23 (continued)
Shears, Moments and Deflections

15. BEAM FIXED AT BOTH ENDS — UNIFORMLY DISTRIBUTED LOADS



Total Equiv. Uniform Load .... $= \frac{2wl}{3}$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

M_{max} (at ends) .... $= \frac{wl^2}{12}$

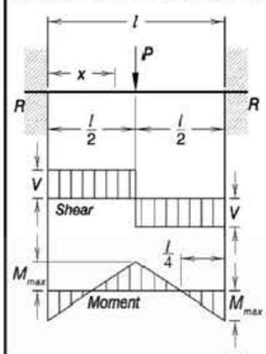
M_1 (at center) .... $= \frac{wl^2}{24}$

M_x .... $= \frac{w}{12}\left(6lx - l^2 - 6x^2\right)$

Δ_{max} (at center) .... $= \frac{wl^4}{384EI}$

Δ_x .... $= \frac{wx^2}{24EI}(l - x)^2$

16. BEAM FIXED AT BOTH ENDS — CONCENTRATED LOAD AT CENTER



Total Equiv. Uniform Load .... $= P$

$R = V$ .... $= \frac{P}{2}$

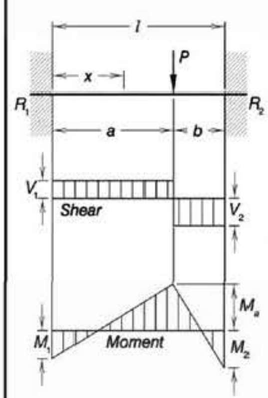
M_{max} (at center and ends) .... $= \frac{Pl}{8}$

M_x (when $x < \frac{l}{2}$) .... $= \frac{P}{8}(4x - l)$

Δ_{max} (at center) .... $= \frac{Pl^3}{192EI}$

Δ_x (when $x < \frac{l}{2}$) .... $= \frac{Px^2}{48EI}(3l - 4x)$

17. BEAM FIXED AT BOTH ENDS — CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ ($= V_{max}$ when $a < b$) .... $= \frac{Pb^2}{l^3}(3a + b)$

$R_2 = V_2$ ($= V_{max}$ when $a > b$) .... $= \frac{Pa^2}{l^3}(a + 3b)$

M_1 ($= M_{max}$ when $a < b$) .... $= \frac{Pab^2}{l^2}$

M_2 ($= M_{max}$ when $a > b$) .... $= \frac{Pa^2b}{l^2}$

M_a (at point of load) .... $= \frac{2Pa^2b^2}{l^3}$

M_x (when $x < a$) .... $= R_1x - \frac{Pab^2}{l^2}$

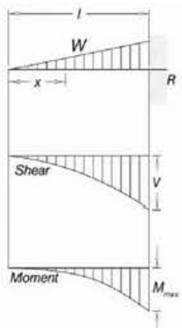
Δ_{max} (when $a > b$ at $x = \frac{2al}{3a + b}$) .... $= \frac{2Pa^3b^2}{3EI(3a + b)^2}$

Δ_a (at point of load) .... $= \frac{Pa^3b^3}{3EIl^3}$

Δ_x (when $x < a$) .... $= \frac{Pb^2x^2}{6EIl^3}(3al - 3ax - bx)$

Table 3-23 (continued)
Shears, Moments and Deflections

18. CANTILEVERED BEAM — LOAD INCREASING UNIFORMLY TO FIXED END



Total Equiv. Uniform Load .... $= \frac{8}{3}W$

$R = V$ .... $= W$

V_x .... $= W\frac{x^2}{l^2}$

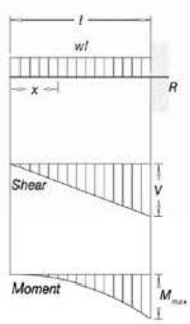
M_{max} (at fixed end) .... $= \frac{Wl}{3}$

M_x .... $= \frac{Wx^3}{3l^2}$

Δ_{max} (at free end) .... $= \frac{Wl^3}{15EI}$

Δ_x .... $= \frac{W}{60EIl^2}\left(x^5 - 5l^4x + 4l^5\right)$

19. CANTILEVERED BEAM — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= 4wl$

$R = V$ .... $= wl$

V_x .... $= wx$

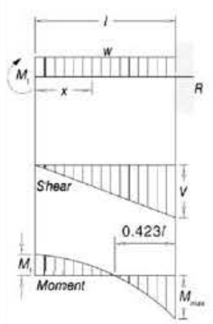
M_{max} (at fixed end) .... $= \frac{wl^2}{2}$

M_x .... $= \frac{wx^2}{2}$

Δ_{max} (at free end) .... $= \frac{wl^4}{8EI}$

Δ_x .... $= \frac{w}{24EI}\left(x^4 - 4l^3x + 3l^4\right)$

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER — UNIFORMLY DISTRIBUTED LOAD



Total Equiv. Uniform Load .... $= \frac{8}{3}wl$

$R = V$ .... $= wl$

V_x .... $= wx$

M_1 (at deflected end) .... $= \frac{wl^2}{6}$

M_{max} (at fixed end) .... $= \frac{wl^2}{3}$

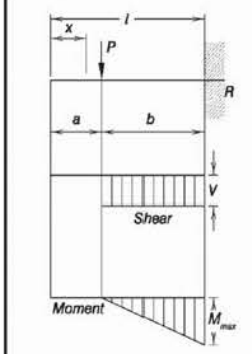
M_x .... $= \frac{w}{6}\left(l^2 - 3x^2\right)$

Δ_{max} (at deflected end) .... $= \frac{wl^4}{24EI}$

Δ_x .... $= \frac{w\left(l^2 - x^2\right)^2}{24EI}$

Table 3-23 (continued)
Shears, Moments and Deflections

21. CANTILEVERED BEAM — CONCENTRATED LOAD AT ANY POINT



Total Equiv. Uniform Load .... $= \frac{8Pb}{l}$

$R = V$ .... $= P$

M_{max} (at fixed end) .... $= Pb$

M_x (when $x > a$) .... $= P(x-a)$

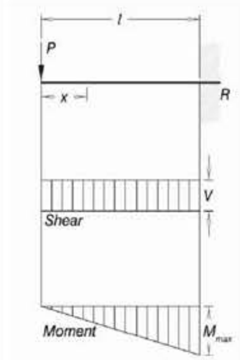
Δ_{max} (at free end) .... $= \frac{Pb^2}{6EI}(3l-b)$

Δ_a (at point of load) .... $= \frac{Pb^3}{3EI}$

Δ_x (when $x < a$) .... $= \frac{Pb^2}{6EI}(3l-3x-b)$

Δ_x (when $x > a$) .... $= \frac{P(l-x)^2}{6EI}(3b-l+x)$

22. CANTILEVERED BEAM — CONCENTRATED LOAD AT FREE END



Total Equiv. Uniform Load .... $= 8P$

$R = V$ .... $= P$

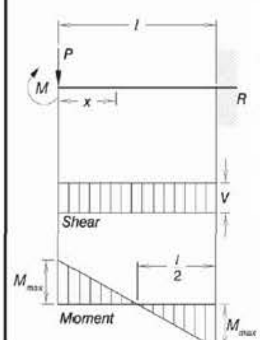
M_{max} (at fixed end) .... $= Pl$

M_x .... $= Px$

Δ_{max} (at free end) .... $= \frac{Pl^3}{3EI}$

Δ_x .... $= \frac{P}{6EI}\left(2l^3 - 3l^2x + x^3\right)$

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER — CONCENTRATED LOAD AT DEFLECTED END



Total Equiv. Uniform Load .... $= 4P$

$R=V$ .... $= P$

M_{max} (at both ends) .... $= \frac{Pl}{2}$

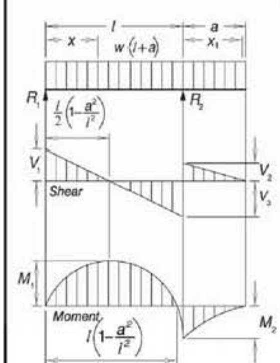
M_x .... $= P\left(\frac{l}{2} - x\right)$

Δ_{max} (at deflected end) .... $= \frac{Pl^3}{12EI}$

Δ_x .... $= \frac{P(l-x)^2}{12EI}(l+2x)$

Table 3-23 (continued)
Shears, Moments and Deflections

24. BEAM OVERHANGING ONE SUPPORT — UNIFORMLY DISTRIBUTED LOAD



$R_1 = V_1$ .... $= \frac{w}{2l}\left(l^2 - a^2\right)$

$R_2 = V_2 + V_3$ .... $= \frac{w}{2l}(l+a)^2$

V_2 .... $= wa$

V_3 .... $= \frac{w}{2l}\left(l^2 + a^2\right)$

V_x (between supports) .... $= R_1 - wx$

V_{x_1} (for overhang) .... $= w(a - x_1)$

M_1 $\left(\text{at } x = \frac{l}{2}\left[1 - \frac{a^2}{l^2}\right]\right)$ .... $= \frac{w}{8l^2}(l+a)^2(l-a)^2$

M_2 (at R_2) .... $= \frac{wa^2}{2}$

M_x (between supports) .... $= \frac{wx}{2l}\left(l^2 - a^2 - xl\right)$

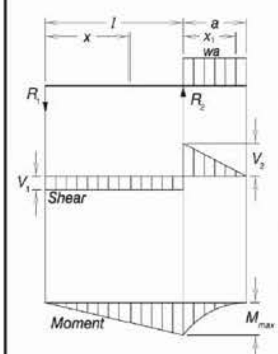
M_{x_1} (for overhang) .... $= \frac{w}{2}(a - x_1)^2$

Δ_x (between supports) .... $= \frac{wx}{24EIl}\left(l^4 - 2l^2x^2 + lx^3 - 2a^2l^2 + 2a^2x^2\right)$

Δ_{x_1} (for overhang) .... $= \frac{wx_1}{24EI}\left(4a^2l - l^3 + 6a^2x_1 - 4ax_1^2 + x_1^3\right)$

NOTE: For a negative value of Δ_x, deflection is upward.

25. BEAM OVERHANGING ONE SUPPORT — UNIFORMLY DISTRIBUTED LOAD ON OVERHANG



$R_1 = V_1$ .... $= \frac{wa^2}{2l}$

$R_2 = V_1 + V_2$ .... $= \frac{wa}{2l}(2l + a)$

V_2 .... $= wa$

V_{x_1} (for overhang) .... $= w(a - x_1)$

M_{max} (at R_2) .... $= \frac{wa^2}{2}$

M_x (between supports) .... $= \frac{wa^2x}{2l}$

M_{x_1} (for overhang) .... $= \frac{w}{2}(a - x_1)^2$

Δ_{max} $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right) = \frac{wa^2l^2}{18\sqrt{3}EI} = 0.0321\frac{wa^2l^2}{EI}$

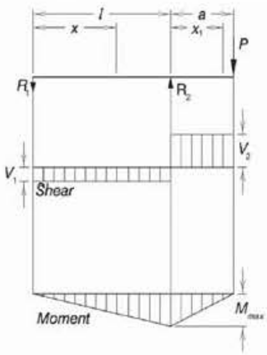
Δ_{max} (for overhang at $x_1 = a$) .... $= \frac{wa^3}{24EI}(4l + 3a)$

Δ_x (between supports) .... $= \frac{wa^2x}{12EIl}\left(l^2 - x^2\right)$

Δ_{x_1} (for overhang) .... $= \frac{wx_1}{24EI}\left(4a^2l + 6a^2x_1 - 4ax_1^2 + x_1^3\right)$

Table 3-23 (continued)
Shears, Moments and Deflections

26. BEAM OVERHANGING ONE SUPPORT — CONCENTRATED LOAD AT END OF OVERHANG



$R_1 = V_1$ .... $= \frac{Pa}{l}$

$R_2 = V_1 + V_2$ .... $= \frac{P}{l}(l+a)$

V_2 .... $= P$

M_{max} (at R_2) .... $= Pa$

M_x (between supports) .... $= \frac{Pax}{l}$

M_{x_1} (for overhang) .... $= P(a - x_1)$

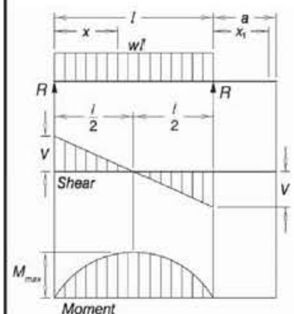
Δ_{max} $\left(\text{between supports at } x = \frac{l}{\sqrt{3}}\right)$ .... $= \frac{Pal^2}{9\sqrt{3}EI} = 0.0642\frac{Pal^2}{EI}$

Δ_{max} (for overhang at $x_1 = a$) .... $= \frac{Pa^2}{3EI}(l+a)$

Δ_x (between supports) .... $= \frac{Pax}{6EIl}\left(l^2 - x^2\right)$

Δ_{x_1} (for overhang) .... $= \frac{Px_1}{6EI}\left(2al + 3ax_1 - x_1^2\right)$

27. BEAM OVERHANGING ONE SUPPORT — UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS



Total Equiv. Uniform Load .... $= wl$

$R = V$ .... $= \frac{wl}{2}$

V_x .... $= w\left(\frac{l}{2} - x\right)$

M_{max} (at center) .... $= \frac{wl^2}{8}$

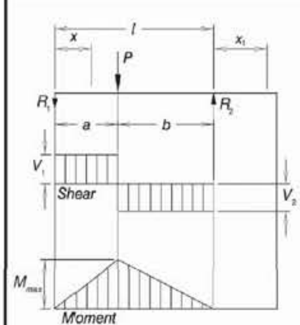
M_x .... $= \frac{wx}{2}(l - x)$

Δ_{max} (at center) .... $= \frac{5wl^4}{384EI}$

Δ_x .... $= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

Δ_{x_1} .... $= \frac{wl^3x_1}{24EI}$

28. BEAM OVERHANGING ONE SUPPORT — CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS



Total Equiv. Uniform Load .... $= \frac{8Pab}{l^2}$

$R_1 = V_1$ (= V_{max} when $a < b$) .... $= \frac{Pb}{l}$

$R_2 = V_2$ (= V_{max} when $a > b$) .... $= \frac{Pa}{l}$

M_{max} (at point of load) .... $= \frac{Pab}{l}$

M_x (when $x < a$) .... $= \frac{Pbx}{l}$

Δ_{max} $\left(\text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b\right)$ .... $= \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIl}$

Δ_a (at point of load) .... $= \frac{Pa^2b^2}{3EIl}$

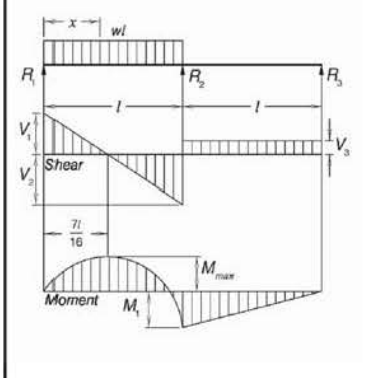
Δ_x (when $x < a$) .... $= \frac{Pbx}{6EIl}\left(l^2 - b^2 - x^2\right)$

Δ_x (when $x > a$) .... $= \frac{Pa(l-x)}{6EIl}\left(2lx - x^2 - a^2\right)$

Δ_{x_1} .... $= \frac{Pabx_1}{6EIl}(l+a)$

Table 3-23 (continued)
Shears, Moments and Deflections

29. CONTINUOUS BEAM — TWO EQUAL SPANS — UNIFORM LOAD ON ONE SPAN



Total Equiv. Uniform Load .... $= \frac{49}{64} wl$

$R_1 = V_1$ .... $= \frac{7}{16} wl$

$R_2 = V_2 + V_3$ .... $= \frac{5}{8} wl$

$R_3 = V_3$ .... $= -\frac{1}{16} wl$

V_2 .... $= \frac{9}{16} wl$

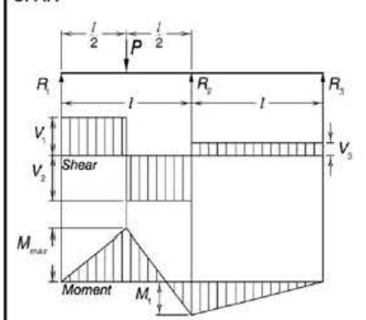
M_{max} (at $x = \frac{7}{16} l$) .... $= \frac{49}{512} wl^2$

M_1 (at support R_2) .... $= \frac{1}{16} wl^2$

M_x (when $x < l$) .... $= \frac{wx}{16}(7l - 8x)$

Δ_{max} (at 0.472 l from R_1) .... $= \frac{0.0092\, wl^4}{EI}$

30. CONTINUOUS BEAM — TWO EQUAL SPANS — CONCENTRATED LOAD AT CENTER OF ONE SPAN



Total Equiv. Uniform Load .... $= \frac{13}{8} P$

$R_1 = V_1$ .... $= \frac{13}{32} P$

$R_2 = V_2 + V_3$ .... $= \frac{11}{16} P$

$R_3 = V_3$ .... $= -\frac{3}{32} P$

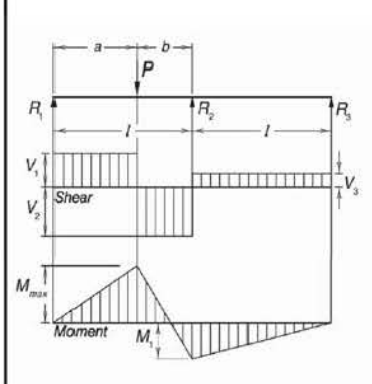
V_2 .... $= \frac{19}{32} P$

M_{max} (at point of load) .... $= \frac{13}{64} Pl$

M_1 (at support R_2) .... $= \frac{3}{32} Pl$

Δ_{max} (at 0.480 l from R_1) .... $= \frac{0.015\, Pl^3}{EI}$

31. CONTINUOUS BEAM — TWO EQUAL SPANS — CONCENTRATED LOAD AT ANY POINT



$R_1 = V_1$ .... $= \frac{Pb}{4l^3}\left(4l^2 - a(l+a)\right)$

$R_2 = V_2 + V_3$ .... $= \frac{Pa}{2l^3}\left(2l^2 + b(l+a)\right)$

$R_3 = V_3$ .... $= -\frac{Pab}{4l^3}(l+a)$

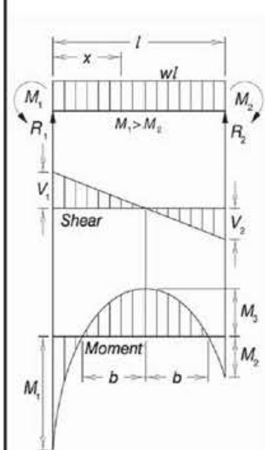
V_2 .... $= \frac{Pa}{4l^3}\left(4l^2 + b(l+a)\right)$

M_{max} (at point of load) .... $= \frac{Pab}{4l^3}\left(4l^2 - a(l+a)\right)$

M_1 (at support R_2) .... $= \frac{Pab}{4l^2}(l+a)$

Table 3-23 (continued)
Shears, Moments and Deflections

32. BEAM — UNIFORMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS



$R_1 = V_1$ .... $= \frac{wl}{2} + \frac{M_1 - M_2}{l}$

$R_2 = V_2$ .... $= \frac{wl}{2} - \frac{M_1 - M_2}{l}$

V_x .... $= w\left(\frac{l}{2} - x\right) + \frac{M_1 - M_2}{l}$

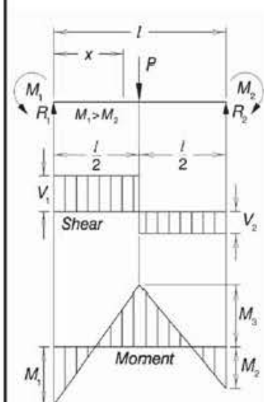
M_3 (at $x = \frac{l}{2} + \frac{M_1 - M_2}{wl}$) .... $= \frac{wl^2}{8} - \frac{M_1 + M_2}{2} + \frac{(M_1 - M_2)^2}{2wl^2}$

M_x .... $= \frac{wx}{2}(l - x) + \left(\frac{M_1 - M_2}{l}\right)x - M_1$

b (to locate inflection points) .... $= \sqrt{\frac{l^2}{4} - \left(\frac{M_1 + M_2}{w}\right) + \left(\frac{M_1 - M_2}{wl}\right)^2}$

$$\Delta_x = \frac{wx}{24EI}\left[x^3 - \left(2l + \frac{4M_1}{wl} - \frac{4M_2}{wl}\right)x^2 + \frac{12M_1}{w}x + l^3 - \frac{8M_1 l}{w} - \frac{4M_2 l}{w}\right]$$

33. BEAM — CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS



$R_1 = V_1$ .... $= \frac{P}{2} + \frac{M_1 - M_2}{l}$

$R_2 = V_2$ .... $= \frac{P}{2} - \frac{M_1 - M_2}{l}$

M_3 (at center) .... $= \frac{Pl}{4} - \frac{M_1 + M_2}{2}$

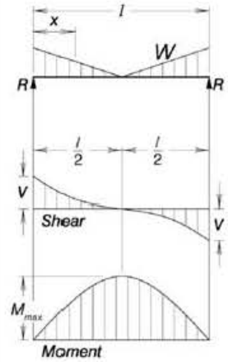
M_x (when $x < \frac{l}{2}$) .... $= \left(\frac{P}{2} + \frac{M_1 - M_2}{l}\right)x - M_1$

M_x (when $x > \frac{l}{2}$) .... $= \frac{P}{2}(l - x) + \frac{(M_1 - M_2)x}{l} - M_1$

Δ_x (when $x < \frac{l}{2}$) $= \frac{Px}{48EI}\left(3l^2 - 4x^2 - \frac{8(l - x)}{Pl}\left[M_1(2l - x) + M_2(l + x)\right]\right)$

Table 3-23 (continued)
Shears, Moments and Deflections

34. SIMPLE BEAM — LOAD INCREASING UNIFORMLY FROM CENTER



Total Equiv. Uniform Load .... $= \frac{2W}{3}$

$R = V$ .... $= \frac{W}{2}$

V_x (when $x < \frac{l}{2}$) .... $= \frac{W}{2}\left(\frac{l-2x}{l}\right)^2$

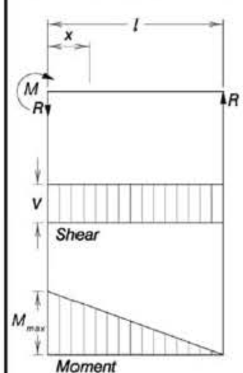
M_{max} (at center) .... $= \frac{Wl}{12}$

M_x (when $x < \frac{l}{2}$) .... $= \frac{W}{2}\left(x - \frac{2x^2}{l} + \frac{4x^3}{3l^2}\right)$

Δ_{max} (at center) .... $= \frac{3Wl^3}{320EI}$

Δ_x (when $x < \frac{l}{2}$) .... $= \frac{W}{12EI}\left(x^3 - \frac{x^4}{l} + \frac{2x^5}{5l^2} - \frac{3l^2x}{8}\right)$

35. SIMPLE BEAM — CONCENTRATED MOMENT AT END



Total Equiv. Uniform Load .... $= \frac{8M}{l}$

$R = V$ .... $= \frac{M}{l}$

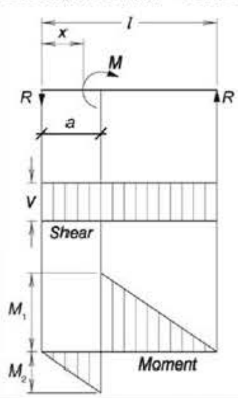
M_{max} .... $= M$

M_x .... $= M\left(1 - \frac{x}{l}\right)$

Δ_{max} (at $x = 0.423\ l$) .... $= 0.0642\frac{Ml^2}{EI}$

Δ_x .... $= \frac{M}{6EI}\left(3x^2 - \frac{x^3}{l} - 2lx\right)$

36. SIMPLE BEAM — CONCENTRATED MOMENT AT ANY POINT



Total Equiv. Uniform Load .... $= \frac{8M}{l}$

$R = V$ .... $= \frac{M}{l}$

M_x (when $x < a$) .... $= Rx$

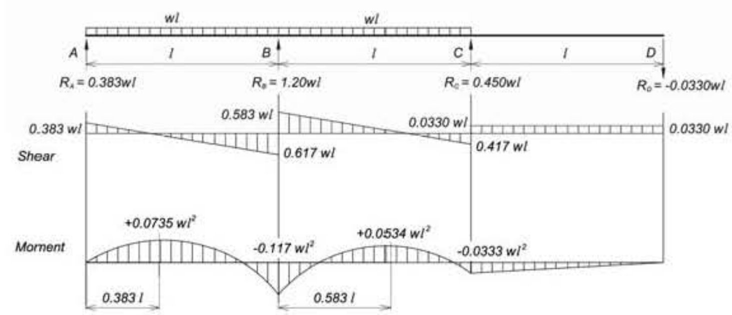
M_x (when $x > a$) .... $= R(l - x)$

Δ_x (when $x < a$) .... $= \frac{M}{6EI}\left[\left(6a - \frac{3a^2}{l} - 2l\right)x - \frac{x^3}{l}\right]$

Δ_x (when $x > a$) .... $= \frac{M}{6EI}\left[3\left(a^2 + x^2\right) - \frac{x^3}{l} - \left(2l + \frac{3a^2}{l}\right)x\right]$

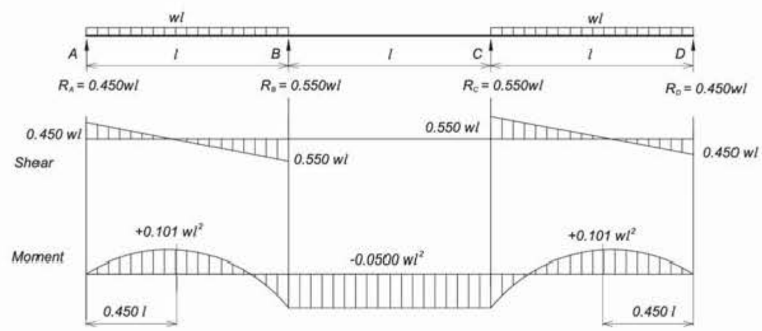
Table 3-23 (continued)
Shears, Moments and Deflections

37. CONTINUOUS BEAM — THREE EQUAL SPANS — ONE END SPAN UNLOADED



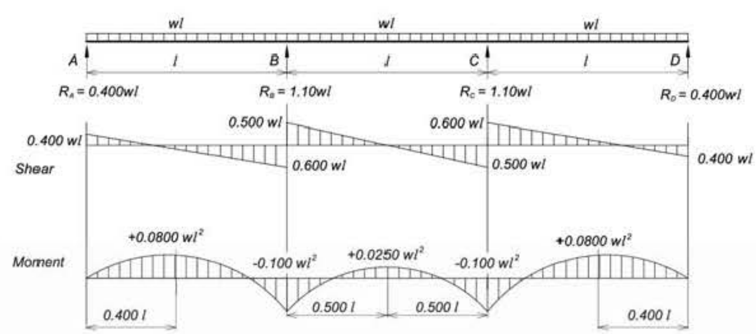
Δ_{max} (0.430l from A) = 0.0059 wl^4/EI

38. CONTINUOUS BEAM — THREE EQUAL SPANS — END SPANS LOADED



Δ_{max} (0.479l from A or D) = 0.0099 wl^4/EI

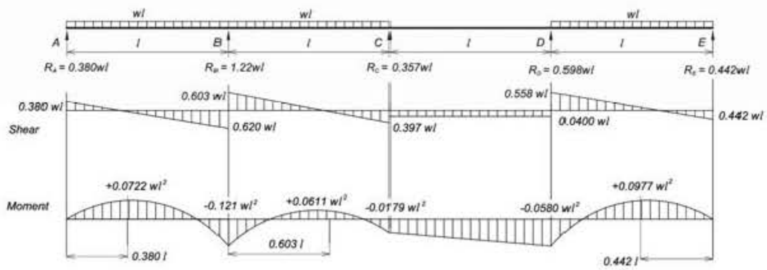
39. CONTINUOUS BEAM — THREE EQUAL SPANS — ALL SPANS LOADED



Δ_{max} (0.446l from A or D) = 0.0069 wl^4/EI

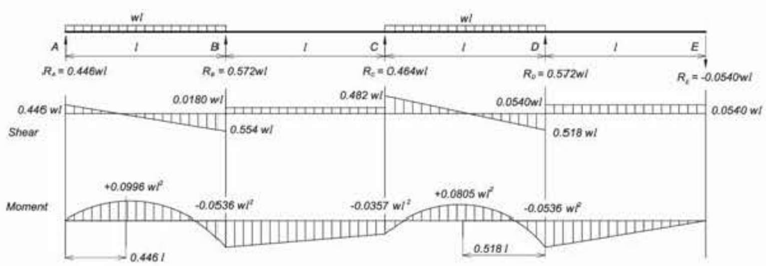
Table 3-23 (continued)
Shears, Moments and Deflections

40. CONTINUOUS BEAM — FOUR EQUAL SPANS — THIRD SPAN UNLOADED



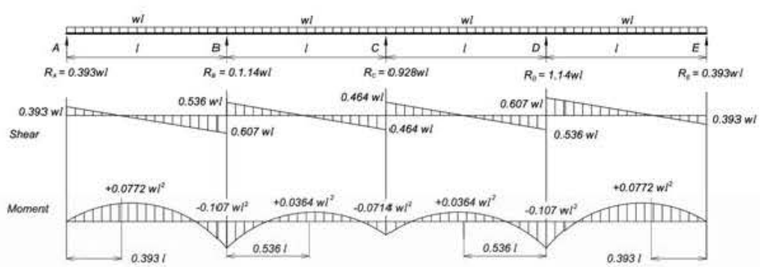
Δ_{max} (0.475 l from E) = 0.0094 wl^4/EI

41. CONTINUOUS BEAM — FOUR EQUAL SPANS — LOAD FIRT AND THIRD SPANS



Δ_{max} (0.477 l from A) = 0.0097 wl^4/EI

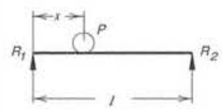
42. CONTINUOUS BEAM — FOUR EQUAL SPANS — ALL SPANS LOADED



Δ_{max} (0.440 l from A and E) = 0.0065 wl^4/EI

Table 3-23 (continued)
Shears, Moments and Deflections

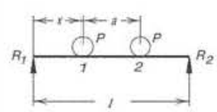
43. SIMPLE BEAM — ONE CONCENTRATED MOVING LOAD



$R_{1\,max} = V_{1\,max}(\text{at } x = 0)$ .... $= P$

$M_{max}\left(\text{at point of load, when } x = \frac{l}{2}\right)$ .... $= \frac{Pl}{4}$

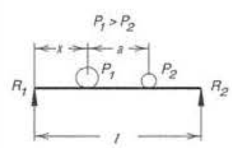
44. SIMPLE BEAM — TWO EQUAL CONCENTRATED MOVING LOADS



$R_{1\,max} = V_{1\,max}(\text{at } x = 0)$ .... $= P\left(2 - \frac{a}{l}\right)$

M_{max}
- $\left[\text{when } a < (2 - \sqrt{2})l = 0.586l \text{ under load 1 at } x = \frac{1}{2}\left(l - \frac{a}{2}\right)\right]$ .... $= \frac{P}{2l}\left(l - \frac{a}{2}\right)^2$
- $\left[\text{when } a > (2 - \sqrt{2})l = 0.586l \text{ with one load at center of span (Case 43)}\right]$ .... $= \frac{Pl}{4}$

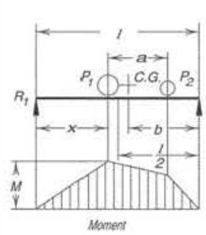
45. SIMPLE BEAM — TWO UNEQUAL CONCENTRATED MOVING LOADS



$R_{1\,max} = V_{1\,max}(\text{at } x = 0)$ .... $= P_1 + P_2\frac{l - a}{l}$

M_{max}
- $\left[\text{under } P_1, \text{ at } x = \frac{1}{2}\left(l - \frac{P_2 a}{P_1 + P_2}\right)\right]$ .... $= (P_1 + P_2)\frac{x^2}{l}$
- [M_{max} may occur with larger load at center of span and other load off span (Case 43)] .... $= \frac{P_1 l}{4}$

GENERAL RULES FOR SIMPLE BEAMS CARRYING MOVING CONCENTRATED LOADS



The maximum shear due to moving concentrated loads occurs at one support when one of the loads is at that support. With several moving loads, the location that will produce maximum shear must be determined by trial.

The maximum bending moment produced by moving concentrated loads occurs under one of the loads when that load is as far from one support as the center of gravity of all the moving loads on the beam is from the other support.

In the accompanying diagram, the maximum bending moment occurs under load P_1 when $x = b$. It should also be noted that this condition occurs when the centerline of the span is midway between the center of gravity of loads and the nearest concentrated load.