# 57:020 Mechanics of Fluids \& Transfer Processes <br> Laboratory Experiment \#2 

## STABILITY OF A FLOATING BODY

## Principle

A floating body is said to be stable at its position, if it returns to that position following a small disturbance.

## Introduction

Laboratory experiment 2 is an exercise in hydrostatics. It is designed to demonstrate the stability of a floating cylinder and to familiarize the student with the concept of buoyancy, metacenter, and metacentric height. It is also an experimental verification of the theory presented in the textbook.

The center of the buoyancy ( $C$, the centroid of the displaced volume of fluid) of a floating body depends on the shape of the body and on the position in which it is floating. If the body is disturbed by a small angle of heel, the center of buoyancy changes because the shape of the submerged volume is changed. The point of intersection of the lines of action of the buoyancy force before and after heel is called the metacenter $(M)$ and the distance between the center of gravity ( $G$ ) and $M$, is called the metacentric height (GM, see Fig. 1).


Fig. 1. Metacentric height of a floating body

The expression for the metacentric height $G M$ is

$$
G M=\frac{I_{O O}}{V}-C G
$$

where $I_{O O}$ is the moment of inertia of the waterline area about the axis of disturbance, and $V$ is volume of the displaced liquid. For stability the metacentric height $G M$ must be positive. Stability (restoring force) increases with increasing GM.

The objective of this experiment is to find the metacentric height and asses the stability of the several floating bodies.

## Apparatus

There are three components of the experimental set-up:

1. Large vertically-standing cylinder containing fresh water
2. Small cylinder with a detachable cap at one end
3. Sand and a metric balance to weigh the cylinders

## Procedures

1. Weigh the small cylinder and its cap together $\left(m_{c}\right)$
2. Place the small empty cylinder into the large vertical cylinder containing fresh water and observe that it is unstable.
3. Pour a small amount of sand into the small cylinder (to give some ballast) and note if it is still unstable. Estimate how much of the cylinder is submerged, $h$, and then measure the height of sand, $h_{b}$.
4. Continue adding sand until the small cylinder remains vertical, i.e., stable. Measure the amount of cylinder that is submerged. Make sure that the open end of the small cylinder is capped when stability is reached.
5. Remove the small cylinder and measure its weight. Also measure the height of sand and record your results on the data sheet.

## Measurements

According to procedures described above, measure the following quantities:

| Case | Mass (gm) | $h(\mathrm{~cm})$ | $h_{b}(\mathrm{~cm})$ |
| :---: | ---: | ---: | ---: |
| Fresh( unstable) |  |  |  |
| Fresh (stable) |  |  |  |

## Data Analysis

Figure 2 shows a slender, hollow circular cylinder which has been made stable in the vertical position by means of ballast placed inside. The cylinder floats in a liquid of density, $\rho$, with a depth of immersion, $h$. The center of buoyancy is $C$, and $G_{C}$ and $G_{b}$ are the centers of gravity of the cylinder and ballast, respectively. The center of gravity of the cylinder/ballast is $G$ and its metacenter is $M$. Other dimensions are shown in the figure. The masses of the cylinder and ballast, $m_{c}$ and $m_{b}$, will be measured during the experiment.


Fig. 2. Hollow cylinder with ballast.

Verify that:

1. The formula for $h$ in terms of the cylinder radius, the density of the liquid $\rho$, and the masses, $m_{c}$ and $m_{b}$,

$$
h=\frac{\left(m_{c}+m_{b}\right)}{\pi R^{2} \rho}
$$

2. The formula for $O G$ in terms of cylinder/ballast dimensions and the masses, $m_{C}$ and $m_{b}$ is

$$
O G=\frac{O G_{c} m_{c}+O G_{b} m_{b}}{\left(m_{c}+m_{b}\right)}=\frac{m_{c} L / 2+m_{b}\left(h_{b} / 2+t\right)}{m_{c}+m_{b}}
$$

3. The expression for the metacentric height, $G M$, is

$$
\begin{aligned}
G M & =C M-C G=C M-(O G-O C)= \\
& =\frac{I_{o o}}{V}-\frac{m_{c} L / 2+m_{b}\left(h_{b} / 2+t\right)}{m_{c}+m_{b}}+h / 2 \\
& =\frac{R^{2}}{4 h}-\frac{m_{c} L / 2+m_{b}\left(h_{b} / 2+t\right)}{m_{c}+m_{b}}+h / 2
\end{aligned}
$$

4. From the immersion, $h$, of the cylinders in the fresh water calculate the density of the fluid in $\mathrm{gm} / \mathrm{cm}^{3}$.
5. Calculate the metacentric height for the fresh in centimeters. Check the stability condition. Record all of your results in the following table.

|  | $R(\mathrm{~cm})$ | $L(\mathrm{~cm})$ | $m_{c}$ | $m_{b}$ | $h(\mathrm{~cm})$ | $h_{b}(\mathrm{~cm})$ | $O G(\mathrm{~cm})$ | $G M(\mathrm{~cm})$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Result |  |  |  |  |  |  |  |  |

## Further Considerations

1. Is there any other equilibrium position of the cylinder except the vertical position?
2. Consider a circular cylinder of homogeneous material of specific gravity, $s=$ 0.5 , length, $L$, and diameter, $D$, stable at the position shown in Fig. 3. Find the formulas for the immersion of the cylinder $h$ at this position.


Fig. 3. Cylinder parallel to the water surface.
3. Consider a cylinder made with a plate thickness, $t=D / 20$, but of a material such that the average value of s is still 0.5 shown in Figure 4. For $L=40 \mathrm{~cm}$ and $D=4 \mathrm{~cm}$, find the position of stable equilibrium and the immersion, $h$.
4. Determine the ranges of $D / L$ for which the positions of the cylinders (vertical and horizontal) are stable.
5. What will happen if we change the density of the fluid, using brine, say, instead of fresh water?


Fig. 4. Hollow cylinder

## References

Roberson, J.A. and Crowe, C.T. (1993). Engineering Fluid Mechanics, 5th edition, Houghton Mifflin, Boston, MA
White, F.M. (1994). Fluid Mechanics, 3rd edition, McGraw-Hill, Inc., New York, NY

# Lab 2 Stability of a Floating Body Data Sheet 

Name $\qquad$ Section $\qquad$
Group Date $\qquad$

Sketch with notation and dimensions

| Case | Mass (gm) | $\mathrm{h}(\mathrm{cm})$ | $\mathrm{h}_{\mathrm{b}}(\mathrm{cm})$ |
| :--- | :--- | :--- | :--- |
| Fresh( unstable) |  |  |  |
| Fresh (stable) |  |  |  |

Results:

| Name | $\mathrm{R}(\mathrm{cm})$ | $\mathrm{L}(\mathrm{cm})$ | $\mathrm{m}_{\mathrm{c}}$ | $\mathrm{m}_{\mathrm{b}}$ | $\mathrm{h}(\mathrm{cm})$ | $\mathrm{h}_{\mathrm{b}}(\mathrm{cm})$ | $\mathrm{OG}(\mathrm{cm})$ | $\mathrm{GM}(\mathrm{cm})$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Result |  |  |  |  |  |  |  |  |

