STRIP METHOD OF SLAB DESIGN

Strip method of slab analysis and design is a lower bound technique in which a slab is divided into strips based on a reasonably assumed load path and load distribution.

Equilibrium is then satisfied throughout the slab.

This method has the following advantages over the other methods:

1. Strip method is applicable for various geometric shapes of slabs such as rectangular, triangular, circular and L-shaped slabs.

2. The method is applicable for slabs having unsupported edges, skewed supports and column supports.

3. Different types of loads, such as uniformly distributed loads, patch loads, linearly varying loads, line loads and concentrated loads, may be considered.

4. Compared with yield line method, strip method is design tool.
5. Use of different amounts of reinforcement in various strips usually makes the design economical.
6. The designer has full control over the distribution of loads and placement of reinforcement.
7. Due to lower bound approach, this method gives results on the safe side.

8. The calculations for slab deflections are easy and straightforward. Cracking may also be minimized if care is taken to select the distribution of loads and moments.

9. Slab with larger openings can also be designed.

10. Strong bands of steel may be used around openings or over columns improving deflections, cracking and economy.

Basic Assumptions

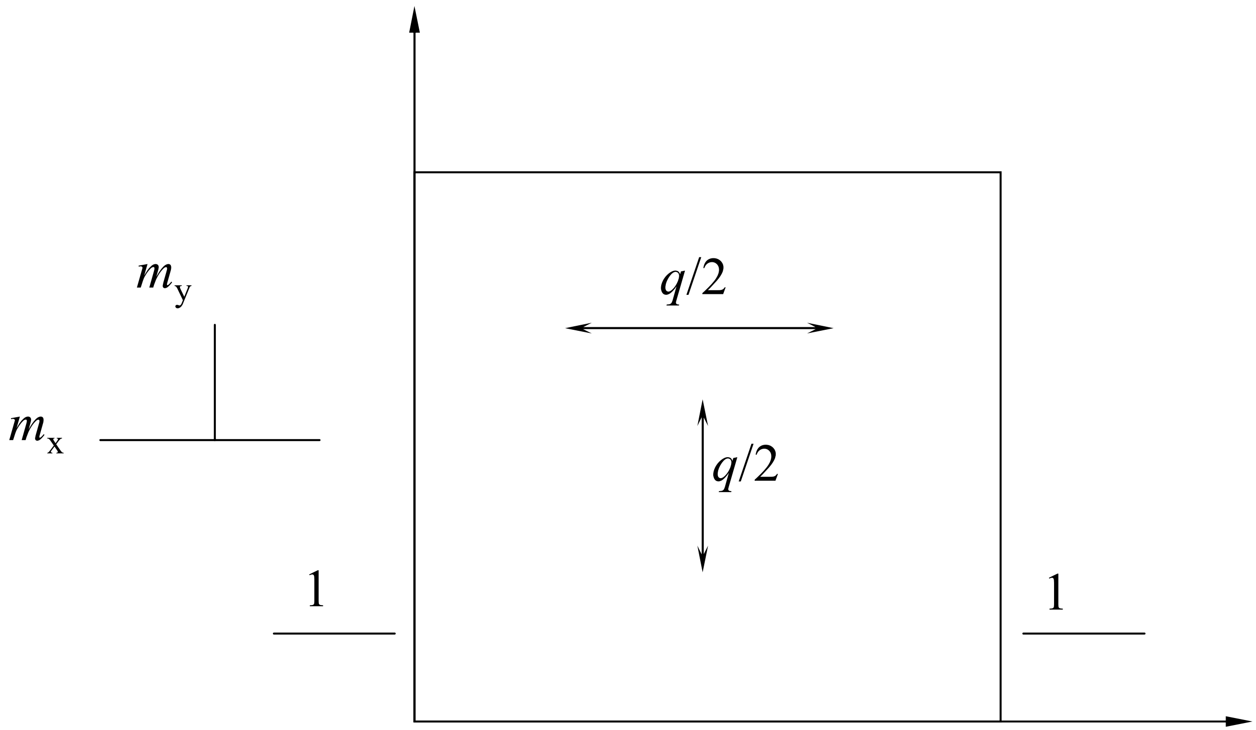
The following two assumptions are made in this method to simplify the solution:

1. The torsional moment capacity is neglected, which gives extra safety in design.

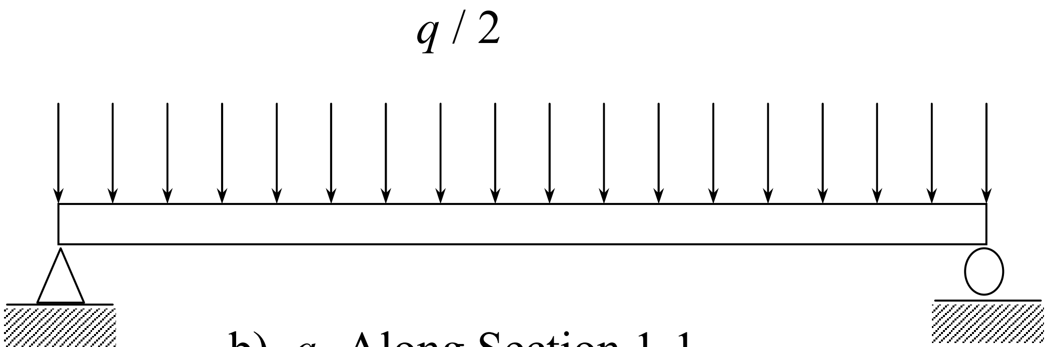
2. The load distribution is reasonably assumed in the start to simplify the analysis. The choice of load transfer is to be selected to get safe and economical design and to avoid excessive cracking and deflections.

Simply Supported Square Slab

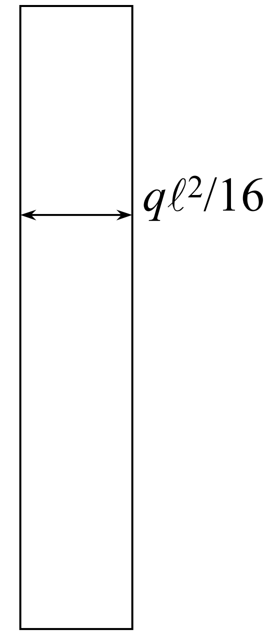
First Load Distribution Pattern



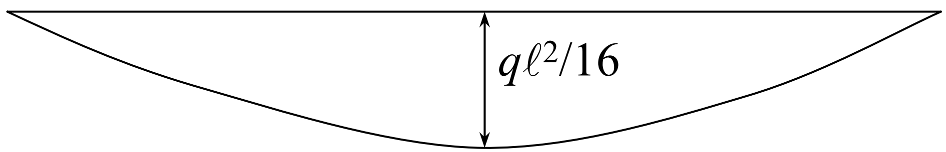
a) Top View of Slab.



b) q_x Along Section 1-1.



d) m_y Across Line $x = \ell/2$.



c) m_y Along Section 1-1.

Fig. 13.36. First Load Distribution of a Square Slab For Strip Method of Analysis.

Let k be the factor of total load carried by the x-direction strips and $1 - k$ be the factor of total load carried by the y-direction strips.

In this case, whole of the slab may be considered as strips in the two perpendicular directions.

The simplest pattern is obtained by assuming that half of the load is distributed in each direction, i.e., the solution is simplified by assuming that $k = 0.5$.

This assumption provides an answer for the required moment capacity equal to $q\ell^2/16$, which is not an economical or workable solution.

The reason is that actually the curvatures are high for the central portions as compared to the end portions.

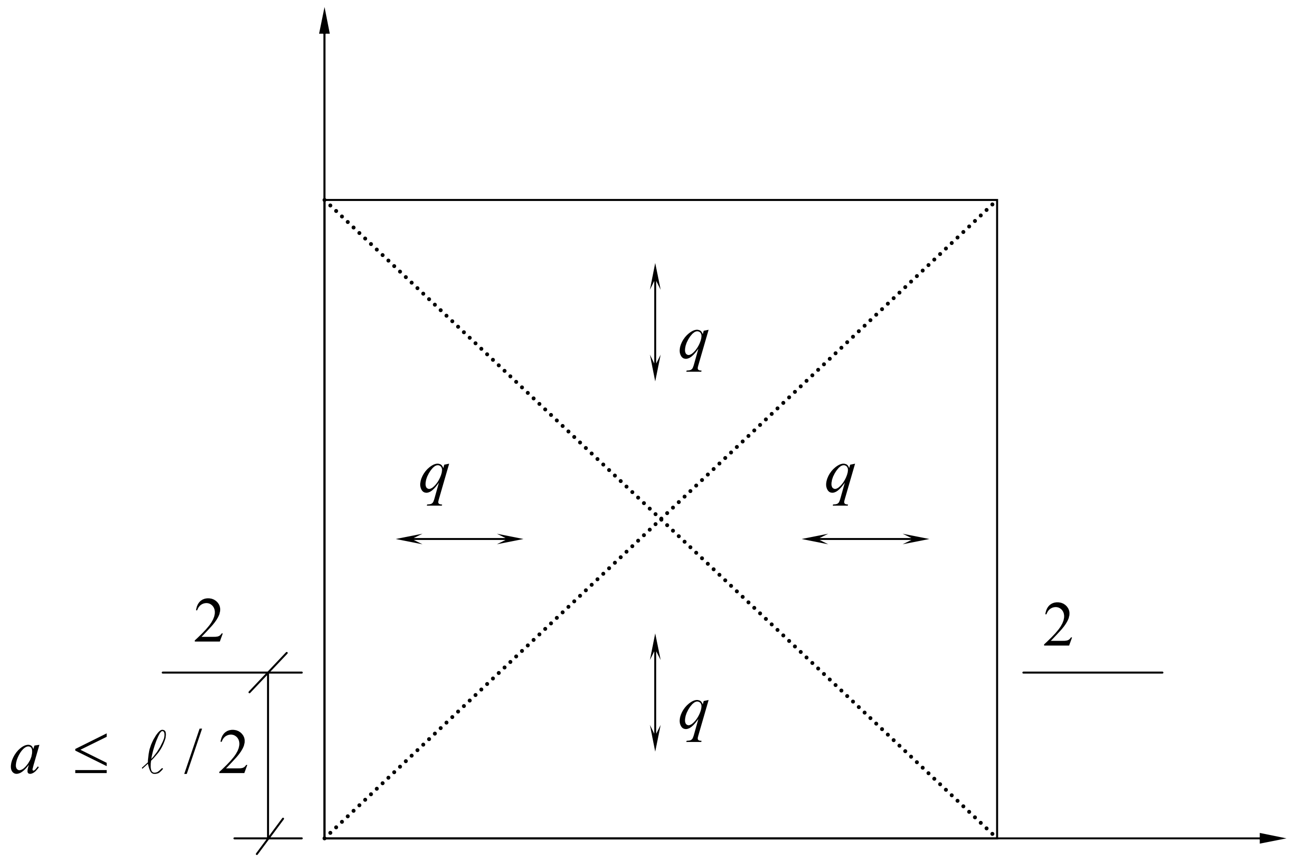
Second Load Distribution Pattern

The slab is considered to be divided into four triangular segments as shown in Fig. 13.37a.

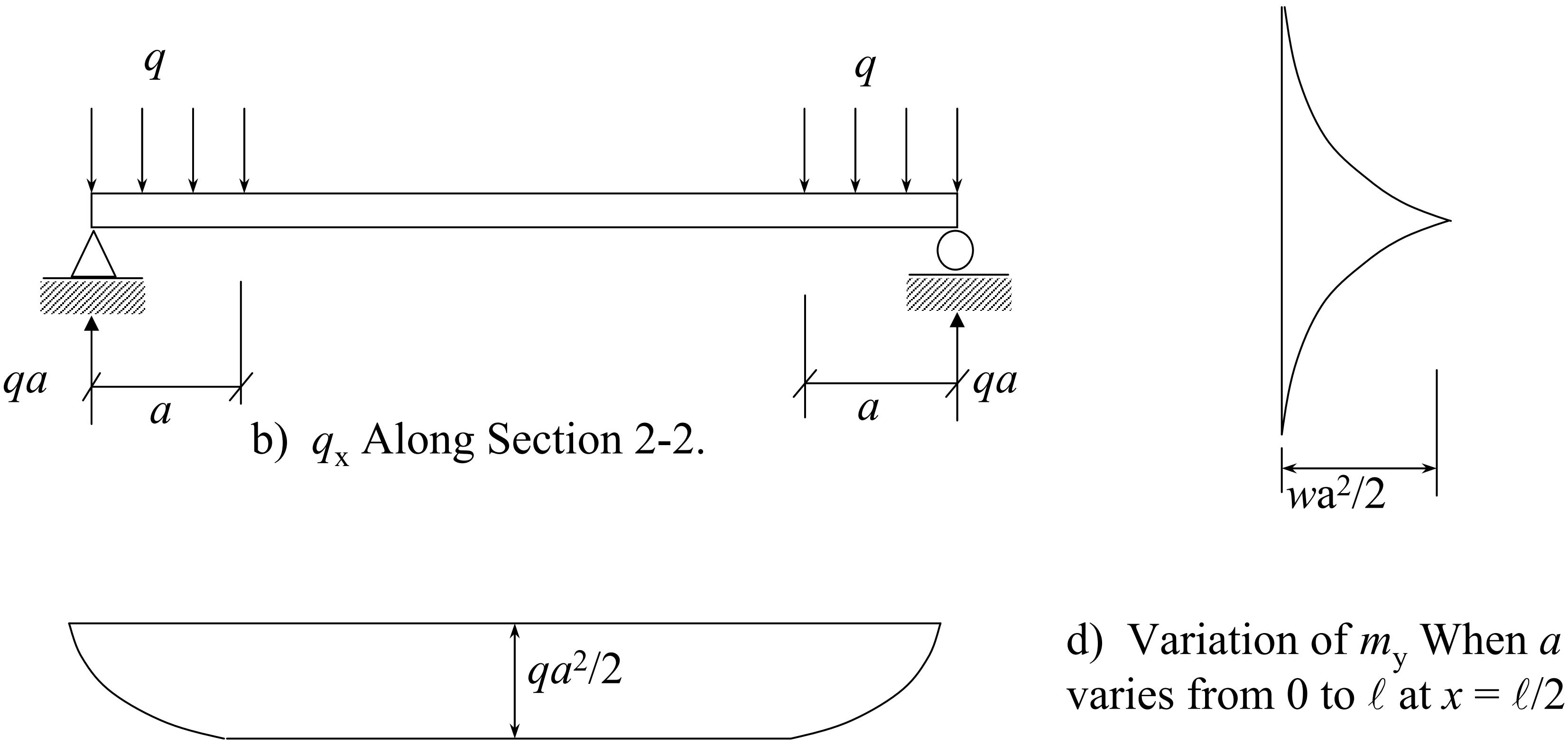
Each slab segment is assumed to transfer load only in a single direction and the *k*-value is either zero or 1 for various segments.

The required maximum bending moment at the center will be $q\ell^2/8$.

The spacing of bars is to be continuously varied along the width, which is practically almost impossible.



a) Top View of Slab.



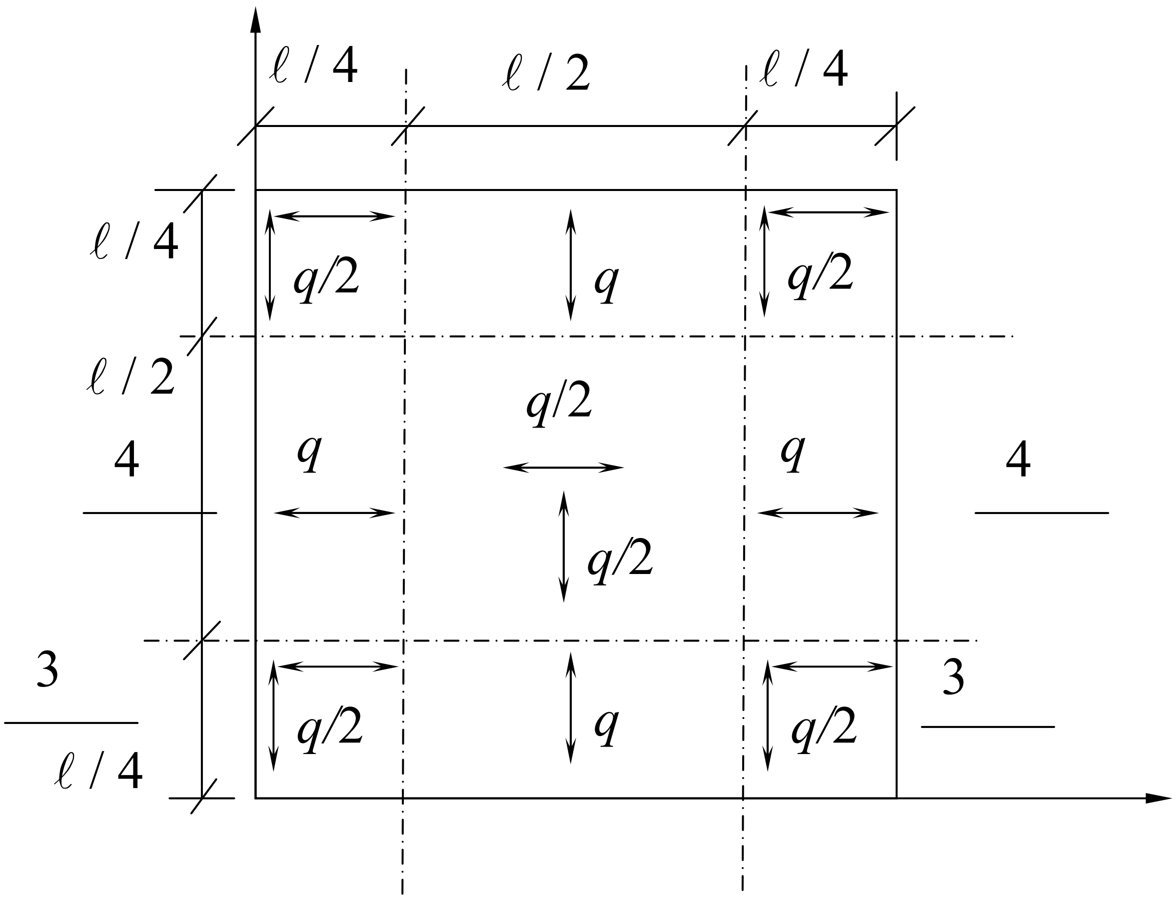
b) q_x Along Section 2-2.

c) m_y Along Section 2-2.

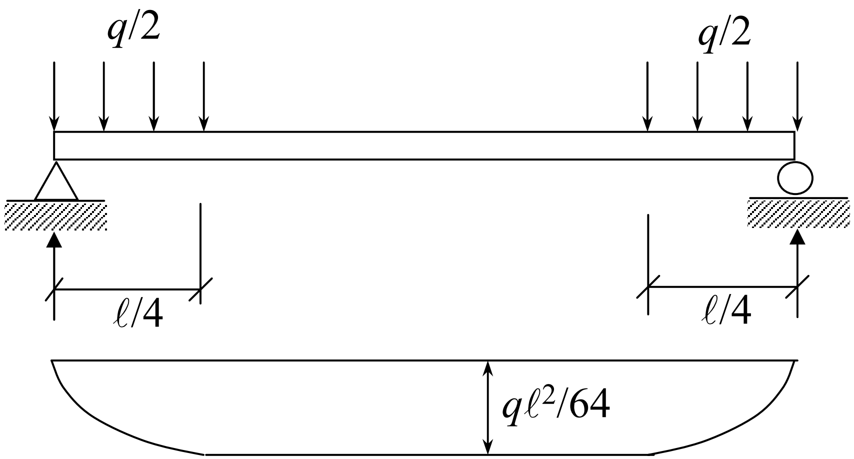
d) Variation of m_y When a varies from 0 to ℓ at $x = \ell/2$.

Fig. 13.37. Second Load Distribution of a Square Slab For Strip Method of Analysis.

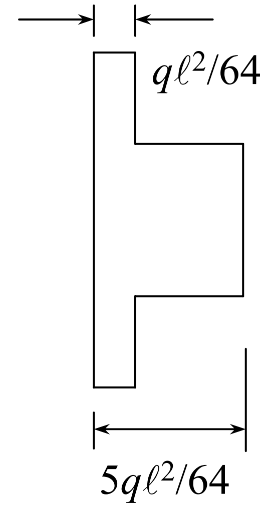
Third Load Distribution Pattern



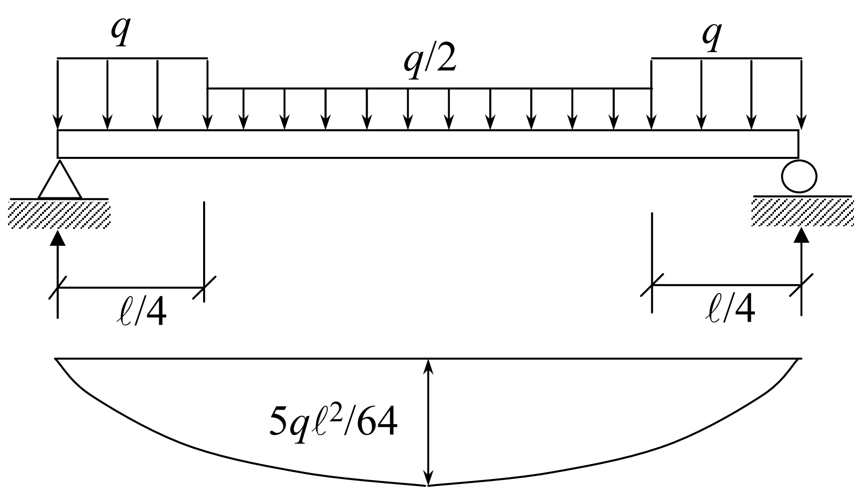
a) Top View of Slab.



b) q_x and m_y Along Section 3-3.



d) Variation of m_y Across $x = \ell/2$.



c) q_x and m_y Along Section 4-4.

Fig. 13.38. Second Load Distribution of a Square Slab For Strip Method of Analysis.

Calculation Of $(m_y)_{max}$ For Section 3-3

End reaction $= q/2 \times \ell/4 = q\ell/8$

$(m_y)_{max} = q\ell/8 \times \ell/4 - q\ell/8 \times \ell/8$

$= q\ell/8 \times 1/8 = q\ell^2/64$

Calculation Of $(m_y)_{max}$ For Section 4-4

End reaction $= q \times \ell/4 + q/2 \times \ell/4 = {}^3/_8 q\ell$

$(m_y)_{max} = {}^3/_8 q\ell \times \ell/2 - q\ell/4 \times (\ell/4 + \ell/8)$

$- q\ell/8 \times \ell/8$

$= q\ell^2 (3/16 - 3/32 - 1/64)$

$= 5q\ell^2/64$

Rectangular Slab With Two Adjacent Edges Continuous And Other Two Simply Supported

The slab may be assumed to be distributed into strips as shown in a-part of the Fig. 13.39, where the assumed distribution of loads is also indicated.

The loading and longitudinal distribution of moments along section 5-5 are given in b-part, while the longitudinal distribution of moments along section 6-6 is shown in c-part of the figure.

Slabs With Continuous Edges

Frequently, in this method, strips that are loaded near the supports and unloaded in the central region are encountered.

It is convenient to keep the unloaded region subjected to a constant moment having zero shear force.

This means that the entire load on a side go to support on that side as a reaction.

By using this treatment, the selection of the positive reinforcement is simplified.

Sum of the absolute values of positive span moment and negative moment at one of the ends is equal to the negative moment if the half span beam acts as cantilever (or only the loaded length on each side) fixed at the corresponding end.

This means that knowing the cantilever moments and selecting the span moment, the corresponding support moment may be evaluated.

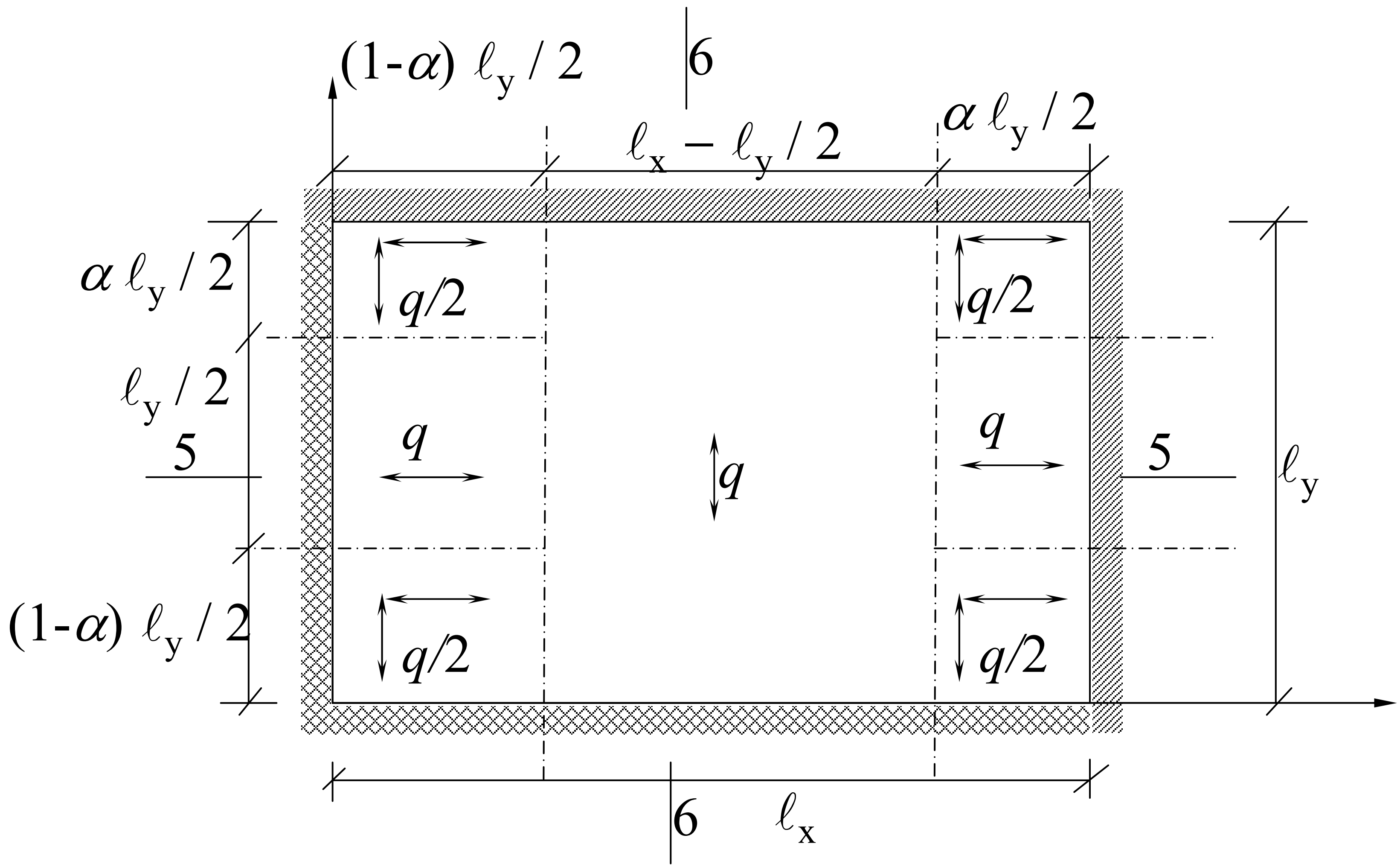
As a general rule, the support moment should be kept equal to about 1.5 to 2.5 times the span moment in the same strip.

For longer, largely unloaded and having more than minimum reinforcement strips, a ratio of support to span moment of 3 to 4 may be used.

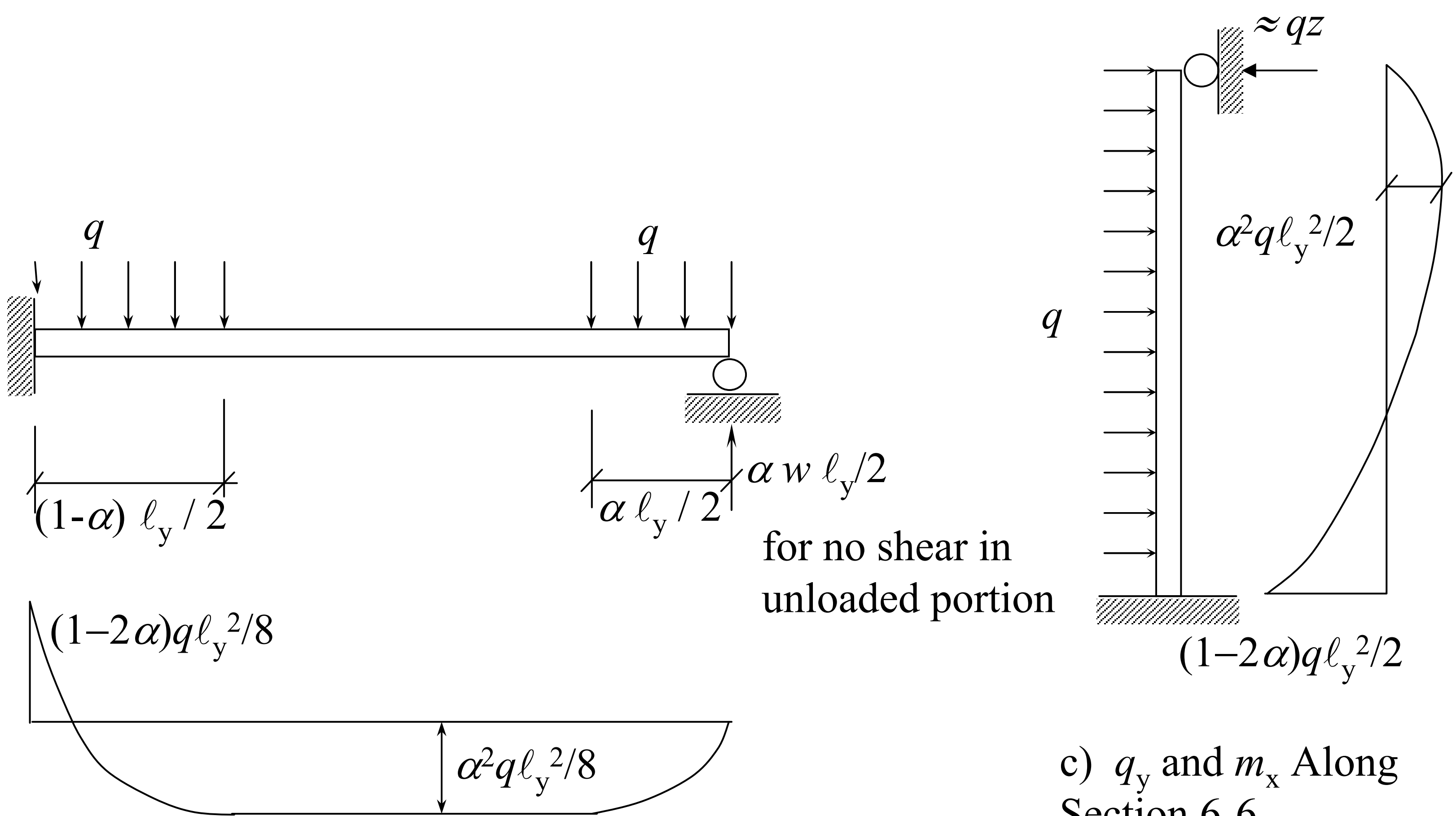
If opposite edges of a slab are fixed and simply supported, the distance of discontinuity lines of the segments from the fixed end is considered larger than the distance from the simply supported end.

The distance towards simply supported end may be taken equal to $\alpha \ell_s / 2$ (ℓ_s is the shorter span) and the distance towards the fixed end will then become $(1 - \alpha) \ell_s / 2$.

Value of α is selected less than or equal to 0.5 and its values between 0.35 and 0.39 give corresponding ratios of negative to positive moments of 2.45 to 1.45 (desired range).



a) Top View of Slab.



b) q_x and m_y Along Section 5-5.

c) q_y and m_x Along Section 6-6.

Fig. 13.39. Rectangular Slab For Strip Method of Analysis.

The maximum positive moment in the x-direction middle strip is:

$$m_{yp} = q\alpha\,\ell_y/2 \times \alpha\,\ell_y/2 - q\alpha\,\ell_y/2 \times \alpha\,\ell_y/4$$
$$= q\alpha\,\ell_y/2\,(\alpha\,\ell_y/4) = \alpha^2 q\,\ell_y^{\,2}/8$$

The cantilever moment at the left support is:

$$m_{c\ell} = q(1-\alpha)\,\ell_y/2 \times (1-\alpha)\,\ell_y/4$$
$$= (1-\alpha)^2 q\,\ell_y^{\,2}/8$$

The negative moment at the left support is:

$$
\begin{aligned}
m_{\ell\mathrm{n}} &= m_{\mathrm{c}\ell} - m_{\mathrm{yp}} \\
&= (1-\alpha)^2\, q\, \ell_{\mathrm{y}}^{\,2}/8 - \alpha^2\, q\, \ell_{\mathrm{y}}^{\,2}/8 \\
&= (1 - 2\alpha)\, q\, \ell_{\mathrm{y}}^{\,2}/8
\end{aligned}
$$

The ratio of negative to positive moment is:

$$
\frac{m_{\ell n}}{m_{yp}} = \frac{1-2\alpha}{\alpha^2}
$$

The moments in the x-direction edge strips are one-half of the middle strip moments. Similarly, y-direction outer strip results are exactly same as those for the x-direction outer strips.

For the y-direction inner strip, the ratio between the positive and the negative moments for the x-direction may be employed giving:

Cantilever moment $= q\,\ell_y^{\,2}/2$

$m_{xp} = \alpha^2 q\,\ell_y^{\,2}/2$ and $m_{xn} = (1 - 2\alpha)\,q\,\ell_y^{\,2}/2$

Propped reaction $\approx q \times z$

Positive moment $= \alpha^2 q\,\ell_y^{\,2}/2$

$= qz \times z - qz \times z/2$

$= qz^2/2$

$\therefore \quad z = \alpha\,\ell_y$

Example 13.11: Design a typical interior panel shown in Fig. 13.40 by using strip method, having clear spans as indicated in the figure. Service live load is 600 kg/m^2 and the superimposed dead load is 200 kg/m^2. f_c'=20 MPa and f_y = *420 MPa.* Use a ratio of support moment to span moment of 2.0.

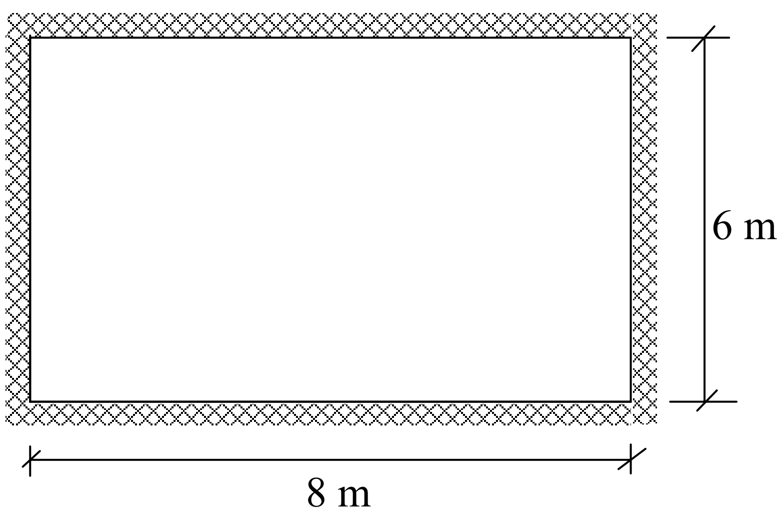


Fig. 13.40.
Rectangular Slab For Example 13.11.

For stiff supports, $$h_{\min} = \frac{\ell_n (0.8 + f_y / 1400)}{36 + 9\beta}$$

where ℓ_{n} = clear longer span = 8000 mm

and $\beta = \frac{\text{clear long span}}{\text{clear short span}} = 8/6 = 1.33$

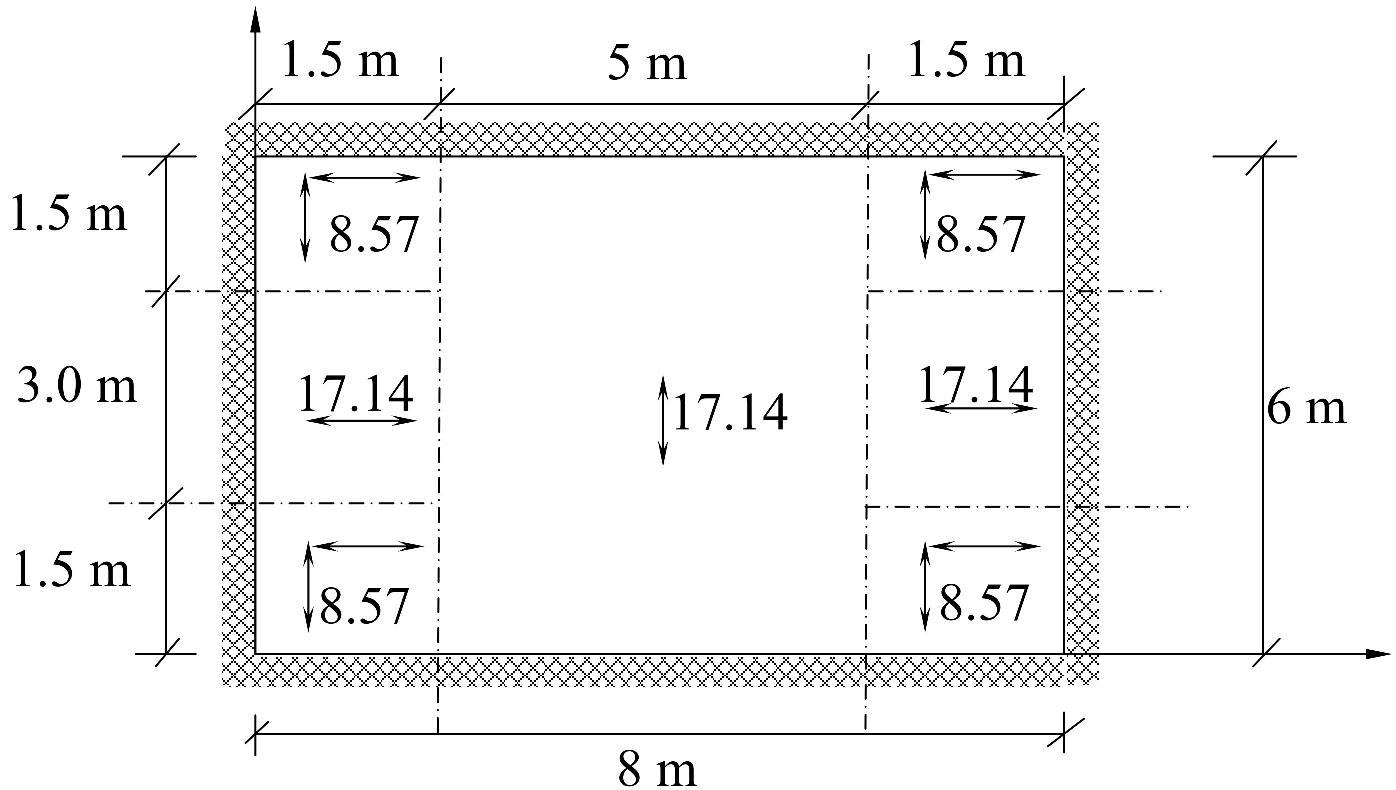
$$h_{\min} = \frac{8000\,(0.8 + 420/1400)}{36 + 9 \times 1.33}$$

$$= 184 \text{ mm} \,(\text{say } 190 \text{ mm})$$

Self weight = $0.19 \times 2400 = 456$ kg/m^2

$q_{\mathrm{d}} = 200 + 456 = 656$ kg/m^2

$$q_u = (1.2 \times 656 + 1.6 \times 600) \times 9.81 / 1000$$
$$= 17.14 \text{ kN/m}^2$$



a) Assumed Load Distribution.

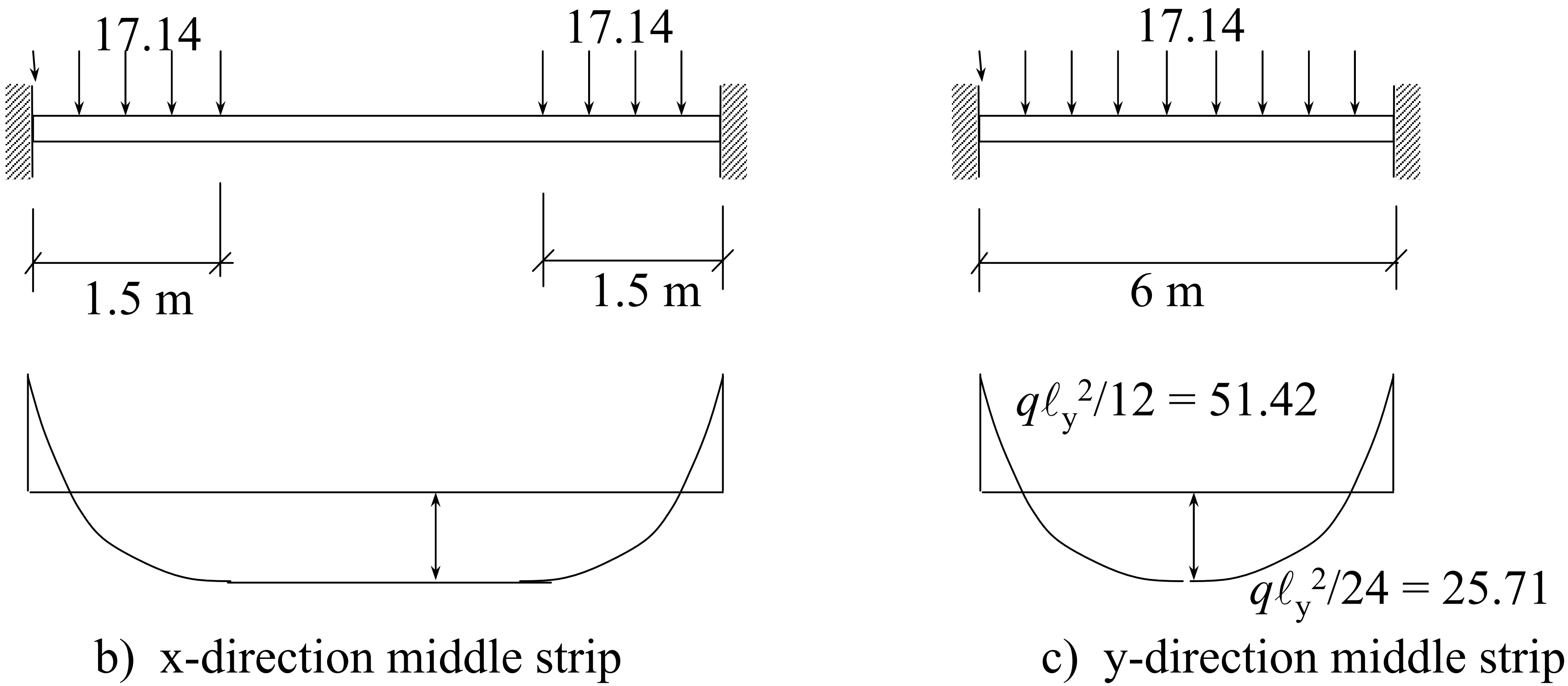


Fig. 13.41. Load And Moment Distribution For Strip Method of Analysis.

X-Direction Middle Strip

Cantilever moment $= 17.14 \times 1.5^2 / 2$

$= 19.28$ kN-m/m

This moment is to be divided in the ratio of 2 negative : 1 positive.

Negative moment $= {}^2/_3 \times 19.28 = 12.86$ kN-m/m

Positive moment $= {}^1/_3 \times 19.28 = 6.43$ kN-m/m

X- And Y- Direction Edge Strips: The moments are half of that for the x-direction middle strip.

Negative moment $= 12.86 / 2 = 6.43$ kN-m/m

Positive moment $= 6.43 / 2 = 3.22$ kN-m/m

Y-Direction Middle Strip

Cantilever moment about left or right support considering half span $= 17.14 \times 3.0^2 / 2 = 77.13$ kN-m/m

This moment is to be divided in the ratio of 2 negative : 1 positive.

Negative moment $= {}^2/_3 \times 77.13 = 51.42$ kN-m/m

Positive moment $= {}^1/_3 \times 77.13 = 25.71$ kN-m/m

$A_{s,min} = 0.0018 \times 1000 \times 190 = 342$ mm²/m

(#10 @ 200 mm c/c)

d in short direction $= 190 - 20 - 13 / 2 \approx 163$ mm

d in long direction $= 190 - 20 - 13 - 10 / 2 = 152$ mm

$$\rho \text{ for } A_{s,min} = \frac{342}{1000 \times 152} = 0.00225$$

From tables, $\dfrac{\phi_b M_n}{bd^2} = 0.8269$

$$\phi_b M_n = 0.8269 \times 1000 \times 152^2 / 10^6$$
$$= 19.10 \; kN\text{-}m/m$$

This moment capacity is sufficient for all the moments except the y-direction middle strip.

Further, all selected spacing are less than 2h.

For M_u = 51.42 kN-m/m,

$$\frac{M_u}{bd^2} = \frac{51.42 \times 10^6}{1000 \times 163^2} = 1.9353, \ \rho = 0.0055$$

A_s = 897 mm^2/m (#13@140mm c/c)

For M_u = 25.71 kN-m/m,

$$\frac{M_u}{bd^2} = \frac{25.71 \times 10^6}{1000 \times 163^2} = 0.9677, \ \rho = 0.0027$$

A_s = 440 mm^2/m (#13@280mm c/c)

Extension Of X-Direction Negative Steel

The distance, x, from the left end up to the point of inflection can be determined as follows:

$$-12.86 + 17.14 \times 1.5 \times x - 17.14 \times x^2 / 2 = 0$$

$$x = 0.634 \text{ m}$$

The extension beyond the point of inflection must be larger of $d = 143$ mm and $12d_b = 120$ mm. Hence actual bar extension from face of support is equal to $0.634 + 0.143 = 0.777$ m (say 0.8m).

Extension Of Y-Direction Middle Strip Negative Steel

The distance, *x*, from any end up to the point of inflection can be determined as follows:

$$-51.42 + 17.14 \times 3 \times x - 17.14 \times x^2 / 2 = 0$$

$$x = 1.268 \text{ m}$$

$d = 143$ mm and $12d_b = 156$ mm. Hence actual bar extension from face of support is equal to $1.268 + 0.156 = 1.424$ m (say 1.5m).

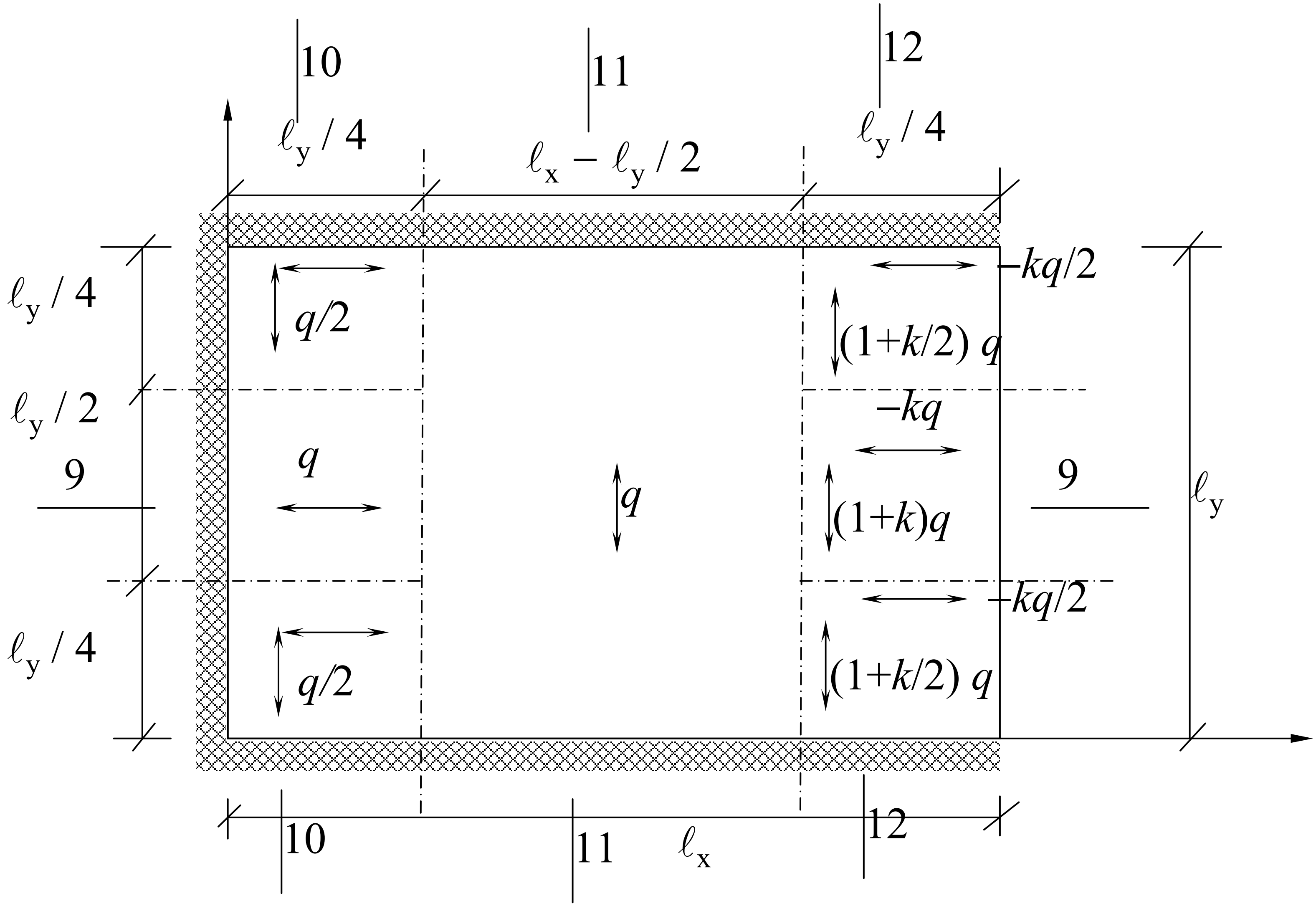
Slabs With Shorter Unsupported Edges

A wide and concealed beam, called ***strong band***, is provided along the unsupported edge.

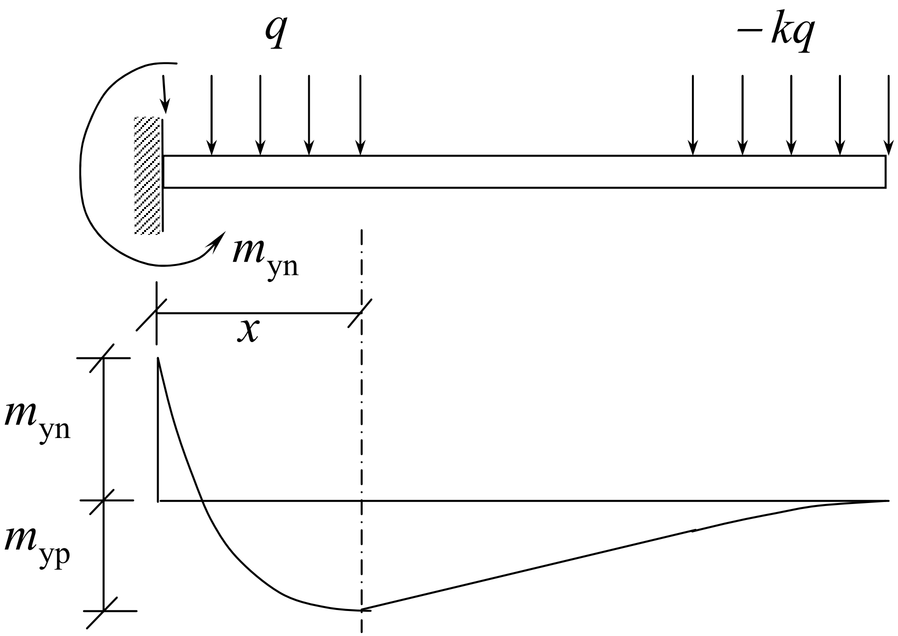
This band resists some additional load from the perpendicular strips.

The additional load in the direction parallel to the unsupported edge is taken as $k \times q$.

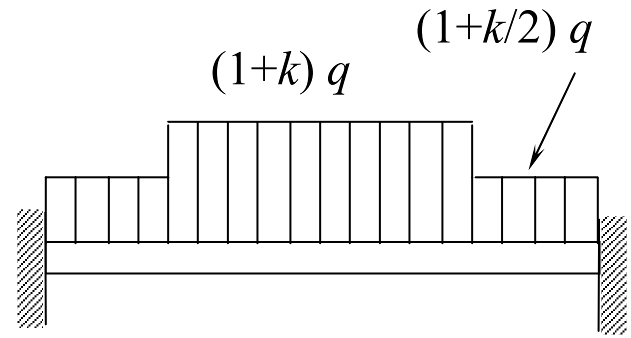
In the perpendicular direction, an opposite load – $k \times q$ is considered for the same portion to keep the resultant load equal to the original value.



a) Top View of Slab.



b) q_x and m_y Along Section 9-9.



c) q_x Along Section 12-12.

Fig. 13.42. Rectangular Slab With Free Shorter Edge.

Considering a middle strip along *x*-direction, as shown in Fig. 13.42, we have:

$$- m_{\mathrm{yn}} + q\,\frac{(\ell_4/4)^2}{2} - kq\,(\ell_{\mathrm{y}}/4)(\,\ell_{\mathrm{x}} - \ell_{\mathrm{y}}/8) \;=\; 0$$

$$\frac{kq\ell_y^2}{32}\left(8\frac{\ell_x}{\ell_y} - 1\right) \;=\; \frac{q\ell_y^2}{32}\left(1 - 32\frac{m_{yn}}{q\ell_y^2}\right)$$

$$k \;=\; \frac{1 - 32\dfrac{m_{yn}}{q\ell_y^2}}{8\dfrac{\ell_x}{\ell_y} - 1}$$

Knowing m_{yn} and k, the distance of maximum positive moment from the left end (x) may be found by considering the shear force (taken due to forces on the right hand side) at maximum positive moment point to be zero as follows:

$$
\begin{aligned}
-kq \times \ell_y/4 + q(\ell_y/4 - x) &= 0 \\
x &= \ell_y/4 - k\,\ell_y/4 \\
&= (1-k)\,\ell_y/4
\end{aligned}
$$

Taking moment of loads on the right of the maximum moment section,

$$
\begin{aligned}
m_{yp} &= kq \times \ell_y/4\,(\ell_x - b/4 - b/8) - q(b/4 - x)^2/2 \\
&= kq \times \ell_y/32\,(8\ell_x - 3\ell_y) - q(\,k^2\,b^2/32\,)
\end{aligned}
$$

$$m_{\mathrm{yp}} = kq\ell_{\mathrm{y}}^{2}/32\,(8\ell_{\mathrm{x}} - 3\ell_{\mathrm{y}}) - q(\,k^2\,b^2/32\,)$$

$$m_{\mathrm{yp}} = \frac{kq\ell_y^2}{32}\left(8\frac{\ell_x}{\ell_y} - 3 - k\right)$$

The moments in the x-direction edge strips are one-half of those in the middle strip.

The moments in the middle y-direction strip may be found as follows:

$$\text{Cantilever moment} = \frac{q(\ell_y/2)^2}{2} = q\ell_{\mathrm{y}}^{2}/8$$

Assuming the negative moment to be 2 times the positive moment,

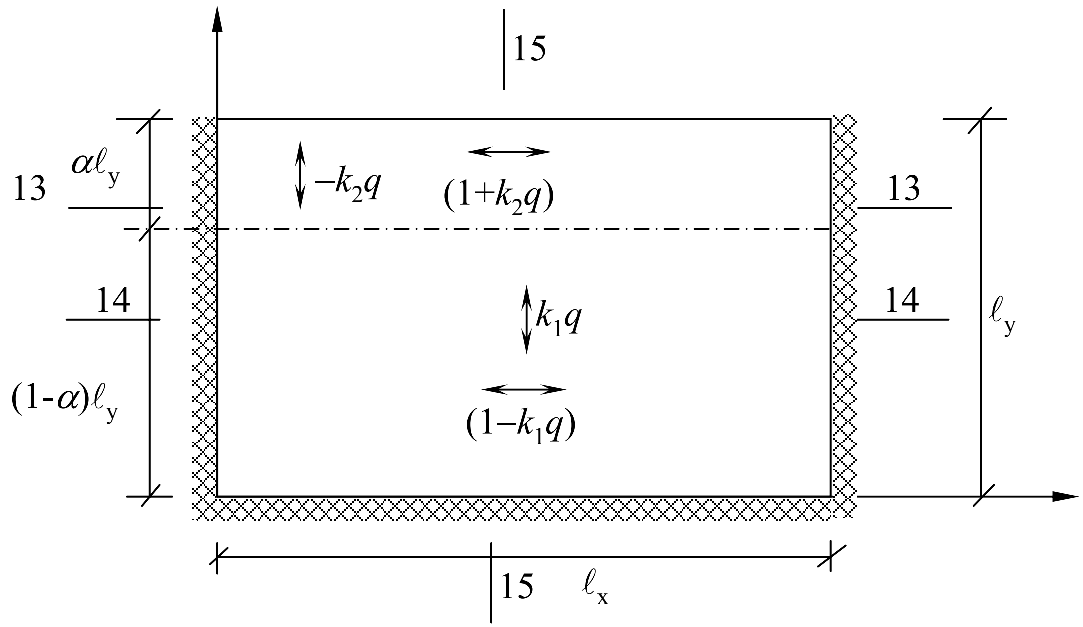
$$m_{xn} = {}^{2}/_{3}\, q\ell_y^{2}/8 = q\ell_y^{2}/12$$
$$m_{xp} = {}^{1}/_{3}\, q\ell_y^{2}/8 = q\ell_y^{2}/24$$

The moments in the left edge strip will be half of the above values.

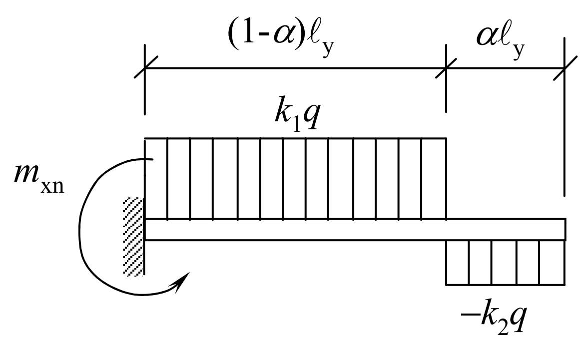
For the edge strip near unsupported end (strong band), the moments may conservatively be found by considering the load $(1 + k)q$ on the full span.

Hence the moments will be $(1 + k)$ times the middle strip moments.

Slabs With Longer Unsupported Edges



a) Top View of Slab.



b) q_y Along Section 15-15.

A larger part of the load still has a natural tendency to go along the shorter path.

However, due to unavailability of a solid support at one edge and the deflections of the strong band, the transfer of load in this direction is reduced to some extent.

The strips along the longer span have to take more share of load as compared to a slab where all the edges are supported.

Even in such circumstances, a significant portion of the load is carried in the shorter direction and the strong band has to withstand more load than the other long strips.

For the y-direction strip along section 15-15, as shown in Fig. 13.43, we have:

$$- m_{\mathrm{xn}} + {}^{1}\!/_{2}\ k_1 q\,(1 - \alpha)^2\, \ell_{\mathrm{y}}^{\,2} - k_2 q\ \alpha \ell_{\mathrm{y}}\,(\,\ell_{\mathrm{y}} - \alpha\,\ell_{\mathrm{y}}\,/2) \qquad = \ 0$$

$$k_2 \quad = \quad \frac{k_1(1-\alpha)^2 - 2m_{xn}\,/\,q\ell_y^2}{\alpha(2-\alpha)}$$

The constant α is reasonably selected to get the strong band designed as singly reinforced section (preferably with lesser steel ratio).

Close to minimum bottom steel is usually provided for positive moment in x-direction strip supported along longer edge and the value of k_1 is accordingly selected.

Strips in the x-direction can easily be analyzed due to udl over the full length.

Along the y-direction, the value of m_{xn} may be selected equal to about half of the free cantilever moment up to the center of the strong band, $^1/_4$ $k_1 q \ell_y^2 (1 - \alpha / 2)^2$.

The k_2 value may then be determined, which is then followed by the calculation of positive moment.

The assumed value of m_{xn} automatically adjusts the load path in the two directions.

Greater value of m_{xn} means less vertical uplift provided by the strong band (lesser k_2).

Example 13.12: A 3.5 × 6 m slab is continuous on three sides and unsupported along one of the longer side. Service live load is 600 kg/m^2 in addition to the self-weight. f_c'=25 MPa and f_y = *420 MPa.* Using strip method, design the slab.

Solution:

For a flat plate, $h_{min} \approx \dfrac{\ell_n}{33}$

where ℓ_n = clear long span = 6000 mm

$$h_{\min} = \frac{6000}{33} = 182 \text{ mm (say 190 mm)}$$

$$\text{Self weight} = 0.19 \times 2400 = 456 \text{ kg/m}^2$$

$$q_{\mathrm{d}} = 456 \text{ kg/m}^2 \quad : \quad q_{\ell} = 600 \text{ kg/m}^2$$

$$q_{\mathrm{u}} = (1.2 \times 456 + 1.6 \times 600) \times 9.81 / 1000$$

$$= 14.79 \text{ kN/m}^2$$

Referring to Fig. 3.44, let $\alpha = 0.2$ to evaluate the strong band width.

For the y-direction strip along main slab, we have:

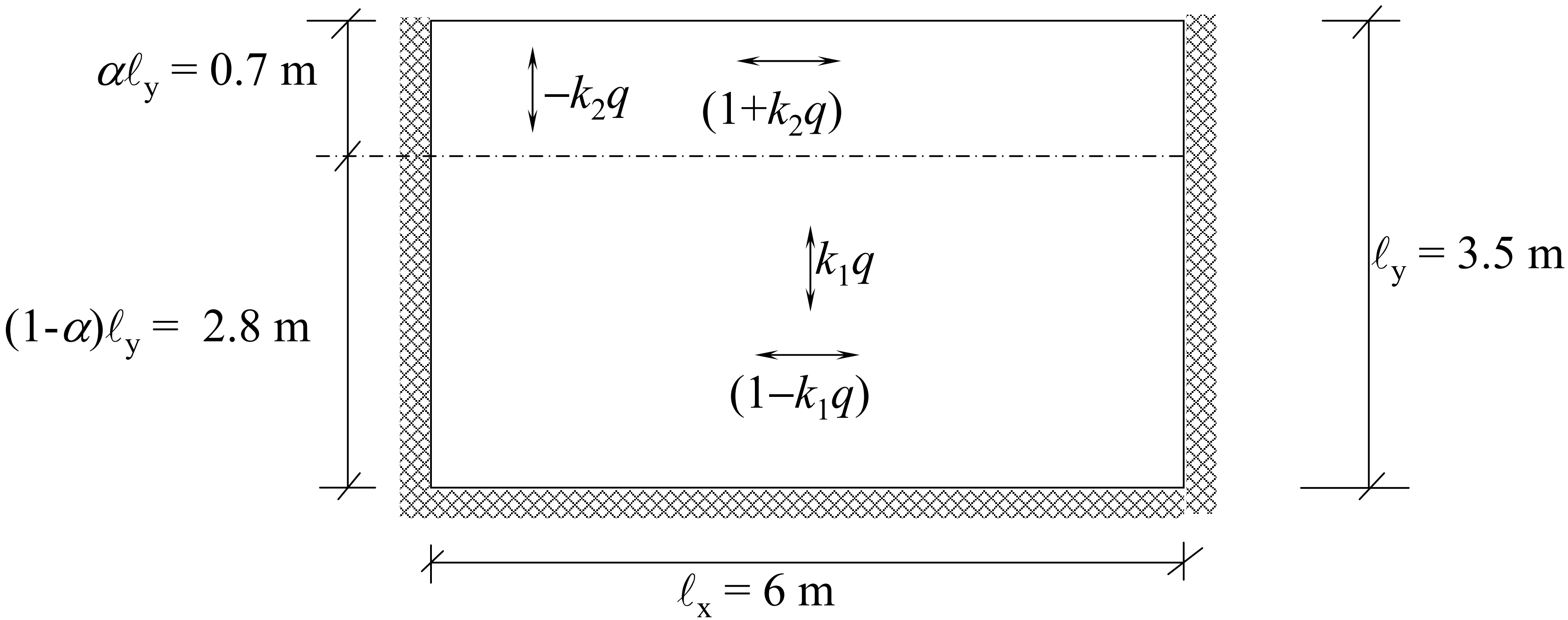


Fig. 13.44. Rectangular Slab of Example 13.12.

$$m_{\mathrm{xn}} = \frac{k_1 q \ell_y^2 (1-\alpha/2)^2}{4}$$

$$= \frac{0.4 \times 14.79 \times 3.5^2 (1-0.2/2)^2}{4} = 14.68 \text{ kN-m/m}$$

$$k_2 = \frac{k_1 (1-\alpha)^2 - 2m_{xn}/q\ell_y^2}{\alpha(2-\alpha)}$$

$$= \frac{0.4 \times (1-0.2)^2 - \dfrac{2 \times 14.68}{14.79 \times 3.5^2}}{0.2 \times (2-0.2)} = 0.261$$

For shear force of the y-direction strip shown in Fig. 13.45, the following may be written:

$$-3.860 \times 0.7 + 5.916 \times (2.8 - y) = 0$$

$$y = 2.343 \text{ m}$$

Shear force at the continuous end is equal to 13.863 kN/m.

The maximum positive moment is,

$$m_{xp} = -14.68 - 5.916 \times 2.343^2 / 2 + 13.863 \times 2.343 = 1.563 \text{ kN-m/m}$$

The distance of point of inflection from the continuous end (y_i) may be found as under:

$$-14.68 - \frac{5.916 y_i^2}{2} + 13.863\, y_i = 0$$

$$y_i = 1.616 \text{ m}$$

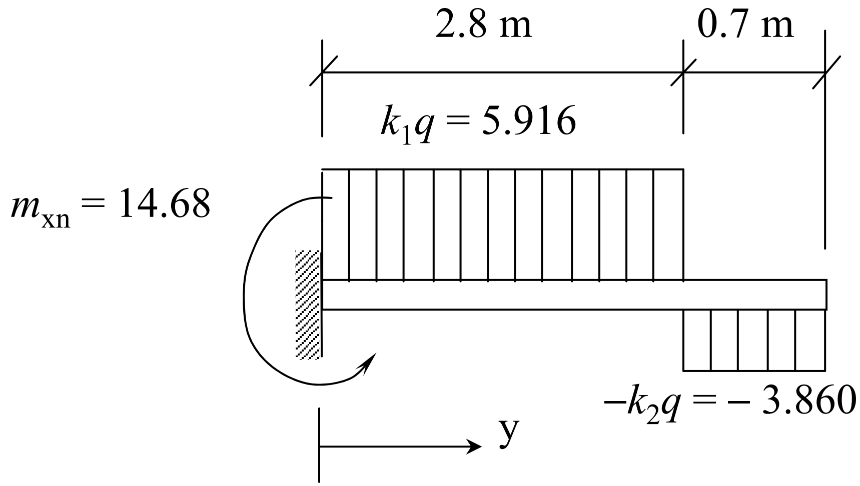


Fig. 13.45. Y-direction Strip for Slab of Fig. 13.44.

For the x-direction strips, the cantilever moment may be evaluated as under:

$$\text{Cantilever moment} = \frac{(1-k_1)q(\ell_x/2)^2}{2}$$

$$= (1-k_1)q\ell_x^2/8$$

Assuming the negative moment to be 2 times the positive moment,

$$m_{yn} = {}^{2}/_{3}\,(1 - k_1)q\ell_x^{\,2}/8 \quad = (1 - k_1)q\ell_x^{\,2}/12$$
$$= (1 - 0.4) \times 14.79 \times 6^2 / 12 = 26.62 \text{ kN-m/m}$$

$$m_{yp} = {}^{1}/_{3}\,(1 - k_1)q\ell_x^{\,2}/8 \quad = (1 - k_1)q\ell_x^{\,2}/24$$
$$= (1 - 0.4) \times 14.79 \times 6^2 / 24 = 13.31 \text{ kN-m/m}$$

Similarly, the moments in the strong band are evaluated as follows:

$$\text{Cantilever moment} = \frac{(1+k_2)q(\ell_x/2)^2}{2}$$
$$= (1 + k_2)q\ell_x^{\,2}/8$$

Assuming the negative moment to be 2 times the positive moment,

$$M_n = {}^{2}/_{3}\,(1 + k_2)q\ell_x^{\,2}/8 = (1 + k_2)q\ell_x^{\,2}/12$$
$$= (1 + 0.261) \times 14.79 \times 6^2 / 12 = 55.95 \text{ kN-m}$$

$$M_p = {}^{1}/_{3}\,(1 + k_2)q\ell_x^{\,2}/8 = (1 + k_2)q\ell_x^{\,2}/24$$
$$= (1 + 0.261) \times 14.79 \times 6^2 / 24 = 27.98 \text{ kN-m}$$

Support reaction for the strong band

$$= \frac{(1 + k_2)q\ell_x}{2} = (1 + 0.261) \times 14.79 \times 6 / 2$$
$$= 55.95 \text{ kN}$$

The distance of point of inflection from the left end (x_i) may be found as under:

$$-55.95 - \frac{18.65x_i^2}{2} + 55.95\,x_i = 0$$

$$x_i = 1.268\text{ m}$$

The location of the point of inflection for the x-direction slab strips will also be as calculated above.

$$A_{s,min} = 0.0018 \times 1000 \times 190$$
$$= 342\text{ mm}^2/\text{m} \quad (\#10 \ @\ 200\text{ mm c/c})$$

d in short direction $= 190 - 20 - 13 / 2$

≈ 163 mm

d in long direction $= 190 - 20 - 13 - 10 / 2$

$= 152$ mm

$f_c{}' = 25$ MPa and $f_y = 420\ MPa.$

$$\rho \text{ for } A_{s,min} = \frac{342}{1000 \times 152} = 0.00225$$

From tables, $\dfrac{\phi_b M_n}{bd^2} = 0.8316$

$\phi_b M_n = 0.8316 \times 1000 \times 152^2 / 10^6 = 19.21\ kN\text{-}m/m$

This moment capacity is sufficient for all the moments except the negative moment for the x-direction strip and the moments for the strong band.

Further, all spacing are to be selected less than $2h$.

x-direction negative moment = 26.62 kN-m/m,

$$\frac{M_u}{bd^2} = \frac{26.62 \times 10^6}{1000 \times 163^2} = 1.0019, \ \rho = 0.0028$$

A_s = 457 mm^2/m (#13 @ 275 mm c/c)

For M_u of strong band = 55.95 kN-m,

$$\frac{M_u}{bd^2} = \frac{55.95 \times 10^6}{700 \times 163^2} = 3.0083, \ \rho = 0.0089$$

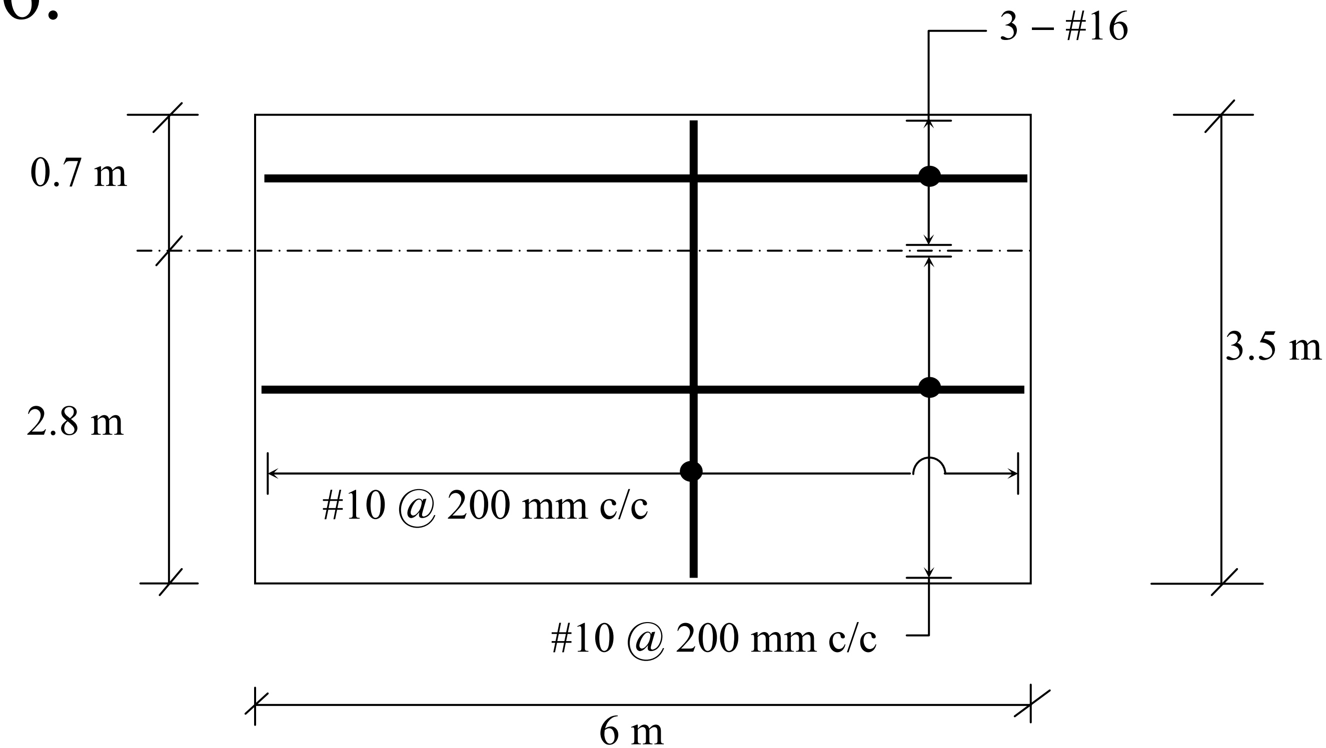
A_s = 1016 mm^2 (6 – #16, @ 117 *mm c/c*)

For M_u of strong band = 27.98 kN-m,

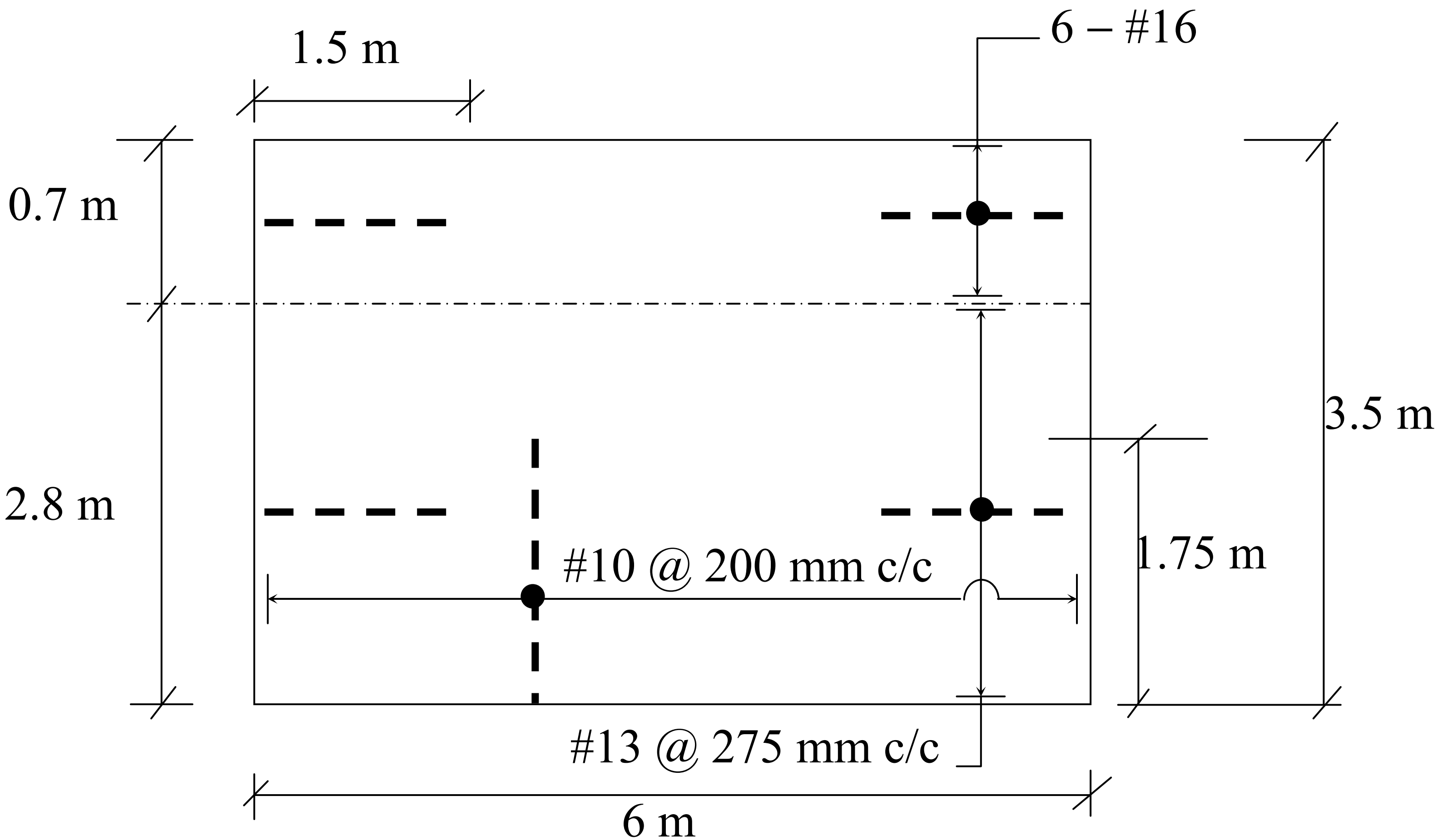
$$\frac{M_u}{bd^2} = \frac{27.98 \times 10^6}{700 \times 163^2} = 1.5044, \ \rho = 0.0043$$

A_s = 491 mm^2 (3–#16, @ 233 *mm c/c*)

Extension of bars equal to $12d_b$ is 120 mm for #10 bars, 156 mm for #13 bars and 180 mm for #16 bars. The resulting reinforcement is shown in Fig. 13.46.



a) Bottom Reinforcement



b) Top Reinforcement

Fig. 13.46. Reinforcement Details for Slab of Fig. 13.44.